Suggested Solutions of Microeconomics Midterm 1
March 31, 2012

1 Part 1 - 5 Points: 2.5 pts each

Please state whether the following sentences are right or wrong and why: (max. 7 lines each)

(i) "Potato is always a Giffen good."

A good is giffen if its demand x increases with its price p, $\frac{dx}{dp} > 0$. The statement is *wrong* because in most of the modern economies, potato is a ordinary good, that is, its demand x decreases with its price p, $\frac{dx}{dp} < 0$. If the price of potato increases, people simply consume more of potato's substitutes such as rice.

To be a Giffen good it has to be the case that the good is inferior and represents a very large share of consumer's expenditures so that the income effect (positive because inferior) is positive and larger than the substitution effect (always negative).

Remarks:

Potato is considered to be giffen in the Irish famine because it represented a substantial share of consumers' expenditures. Therefore, a change in the price of potatoes determined a very large income effect (positive since potatoes are inferior) more than compensating the substitution effect (always negative).

(ii) "If the share of your income spent in food is 0.3 and in housing is 0.7, then the price of housing does not influence the demand for food and vice-versa."

Wrong. Take perfect substitutes preferences for instance. If $MRS_{f,h} = -\frac{p_f}{p_h}$, then the consumer is free to choose between food and housing. She can choose them such that income spent in food and in housing can be 0.3 and 0.7, respectively. However, the price of housing does influence the demand for food and vice-versa.

Remark: We can also find similar cases for other type of preferences, like perfect complements and etc.

Common Mistake: This is Cobb-Douglas preferences and prices does not influence the other good's demand. (*Got still 1 point*). The first part of the statement does not need to hold always.

Bad Mistake: This is Cobb-Douglas and prices does influence the other good's demand.

2 Part 2 - 15 Points: i-vi 1.5 pts each; vii-ix 2 pts each

(i) Imagine that for health reasons what matters is the total amount (in kgs) of raw vegetables that you consume. Assuming that the vegetables that you prefer are carrots and broccoli, represent your preferences for carrots and broccoli by choosing an appropriate utility function, and illustrate graphically the indifference map. Explain briefly.

Carrots and Broccoli are *perfect substitutes* and can be represented by the following utility function $u(b, c) = b + c$, since we only care for the total amount of vegetables, not specifically how many kgs of broccoli or carrots we consume. We are indifferent between consuming one kg of broccoli or one kg of carrot.

Perfect Substitutes preferences are characterized by constant MRS, which is equal to -1 in this case, meaning indifference curves (ICs) are straight lines with slope -1.

Common Mistakes:

The indifference map shows Cobb-Douglas looking, strictly convex IC's.
Giving higher marginal utility to one vegetable, such as, $u(b, c) = 2b + c$.
Cobb-Douglas or Perfect Complements.

(ii) If the price per kg of carrots is $\mathbf{p_c}$= 1, the price per kg of broccoli is $\mathbf{p_b}$= 2, and your income is M = 20, what is your optimal choice?

Since we are indifferent between broccoli and carrot, we consume only the cheaper one, which is carrot in this case. We spend all the money in carrots and consume $\frac{M}{p_c} = 20$. Optimal Choice is then $(b^*, c^*) = (0.20)$.

Graphical argument with IC's and budget constraint (BC) leading to the corner solution, or, writing down the demand functions indicating the result above is also fine.

(iii) Obtain the Engel curve for carrots in case (i) for pc=1 and pb=2.

Engel curve for carrots relates the demand for carrots to income (x and y axis should be M and c). For these prices, we only consume carrots, so the slope of the engel curve has to be its price, which is 1.$M = c$

Common Mistakes:
Not specifying the slope (*got still 1 pt*)
Writing axis wrong or missing them
Drawing the curve wrong.

(iv) The last time you went to the doctor he suggested you to combine the two vegetables so that you eat 0.5 kg of carrots and 0.25 kg of broccoli. How do you represent your preferences in this case? Explain briefly and illustrate graphically the indifference map.

Perfect Complements. Could be represented by the following (*Leontief*) utility functions:

$u(b, c) = \min\{2b; c\}$ or $v(b, c) = \min\{0.5b; 0.25c\}$ and any other monotonic transformation of these utility functions.

We always combine 0.5kg of carrots with 0.25 kg of broccoli. Increasing the consumption of one vegetable without increasing the other does not increase the utility.

The IC's are L-shaped with the kink exactly at the given ratio, that is, for 0.5kg of carrots 0.25 kg of broccoli, for kg of carrots 0.5 kg of broccoli, and so on.

Common Mistakes:
Confusing the magnitudes of the good in the utility function, such as: $u(b, c) = \min\{b; 2c\}$ or $v(b, c) = \min\{0.25b; 0.5c\}$.
Drawing the IC's like Cobb-Douglas, perfect complements
Referring the preference Cobb-Douglas or Perfect Substitutes

(v) If the price per kg of carrots is pc=1, the price per kg of broccoli is pb=2, and your income is M=20, what is your optimal choice? and

(vi) Derive the demand functions for carrots and broccoli for case (iv).

Let's find the demand functions:
Optimality Conditions:

$$2b = c \quad (1)$$
$$p_b b + p_c c = M \quad (2)$$

The condition (1) defines the kinks of the IC's; and the condition (2) guarantees the optimal choice being on the budget line. Substituting (1) into (2) and some algebra give the demand for broccolis:

$$p_b b + p_c 2b = M$$
$$b(.) = \frac{M}{2p_c + p_b} \quad (3)$$

Then, substituting (3) into (1) gives the demand for carrots:

$$c(.) = \frac{2M}{2p_c + p_b} = \frac{M}{p_c + \frac{1}{2}p_b} \tag{4}$$

Substituting the specific values of income and prices into the demand functions give the optimal choice:

$$\left(b^I, c^I\right) = (5, 10)$$

Common Mistakes:
Using $b = 2c$. (*Got some points, if it is consistent with the utility function defined*)
"Using" tangency condition.
Drawing some demand curves in stead of finding the functions analytically.
"Demand functions" depending the other good's quantity/demand.

(vii) If the price of broccoli increases to $\mathbf{p_b}$= 3 how much would you be willing to pay to avoid the price increase? Explain briefly.

This is the definition of the equivalent variation a la Slutsky ($EV^{SLUTSKY}$), the amount of money given in order to get the *final* optimal bundle x^F (after the price change) at the *initial* prices p^I.

Let's find the final optimal bundle after the price change. Substitution the income and new prices into the demand functions gives:

$$\left(b^F, c^F\right) = (4, 8)$$

Applying the definition of the $EV^{SLUTSKY}$, we find how much money the consumer needs.

$$\begin{aligned} p_b^I b^F + p_c^I c^F &= M' \\ 2 \times 4 + 1 \times 8 &= 16 \end{aligned}$$

Then

$$EV^{SLUTSKY} = M' - M = 16 - 20 = -4$$

So, the consumer has to pay 4 euros.

Remark: $EV^{SLUTSKY} = EV^{HICKS}$, since there is no substitution effect with perfect complement preferences. This allows you to find EV^{HICKS} and argue that they give the same amount.

Common Mistakes:
Calculating $CV^{SLUTSKY}$ or change in consumer surplus.

(viii) Imagine that in your own view a well balanced diet is associated to the level of utility that you were enjoying before the price of broccoli went up. In this case, if you were allowed to negotiate some compensation to keep that level of utility what would be the minimum that you were willing to accept? Explain briefly.

This is CV^{HICKS}: the amount of money needed in order to attain the *initial* utility level u^I at the *final* prices p^F. So, the utility level u^{INT} after being compensated M' at the final prices p^F has to be equal to the initial utility level u^I at the initial income M and prices p^I. More formally:

$$u^{INT} = u^I \tag{5}$$

using the utility function $u(b, c) = \min\{2b; c\}$ and demand functions (3) and (4), since the consumer optimizes, (5) becomes:

$$\min\{2 \times \frac{M'}{2p_c + p_b^F}; \frac{2M'}{2p_c + p_b^F}\} = \min\{2 \times 5; 10\} \tag{6}$$

Substituting the prices and leaving $M\prime$ alone:

$$\begin{aligned} \frac{2M'}{2+3} &= 10 \\ M' &= 25 \end{aligned}$$

Then,

$$CV^{HICKS} = M' - M = 25 - 20 = 5$$

Remark: $CV^{HICKS} = CV^{SLUTSKY}$, since there is no substitution effect with perfect complement preferences. This allows you to find EV^{HICKS} and argue that they give the same amount.

Common Mistakes:

Calculating $CV^{SLUTSKY}$ and not arguing that they have to be the same.

(ix) Compare and identify the concepts that you have used to answer (vii) and (viii). Illustrate graphically the results obtained and explain.

We have already said that the concepts are $EV^{SLUTSKY}$ and CV^{HICKS} for (vii) and (viii), respectively. You can see the graph on the slides: Lecture 8, page 5 for instance to have the idea of the comparison between CV and EV. Of course, the IC's has to be L-shaped, since for the consumer broccolis and carrots are perfect complements.

Common Mistakes:

Drawing IC's like Cobb-Douglas, that is, strictly convex IC's.

Some other general concerns:

There were lots of algebraic mistakes

Finding correct answer without writing any previous steps or without following any logical chain. Besides not getting the full grade, this does not give a good impression to the graders. *For instance:* representing the preferences with Cobb-Douglas utility function, but later in the following questions using Leontief utility function and writing down the correct optimal choice and etc.