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1102-Microeconomics

Second Final

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11.30-13.30pm

Warnings

1. Calculators or any other electronic devices are not allowed.
2. No questions are answered during the test.

Honor's Commitment

I declare that I will neither use nor contribute, directly or indirectly, to any illegal procedure or fraud during this test.

Signature: ______________________________

Good Luck!

Solution TOPics

Name: ______________________________ Nº: ________

I
(3)

Suppose a representative firm in a perfectly competitive, constant-cost industry has a cost function

$$TC = 2q^2 + 100q + 100$$

(a) If market demand is given by Q=500-P, where P denotes price, and knowing that there are 12 firms operating in this market, obtain the short-run equilibrium for this market. Illustrate graphically this equilibrium.

a) 2 points

$\max_q \; Pq - TC$

FOC: $P = MgC$

$P = 4q + 100$ → Individual supply

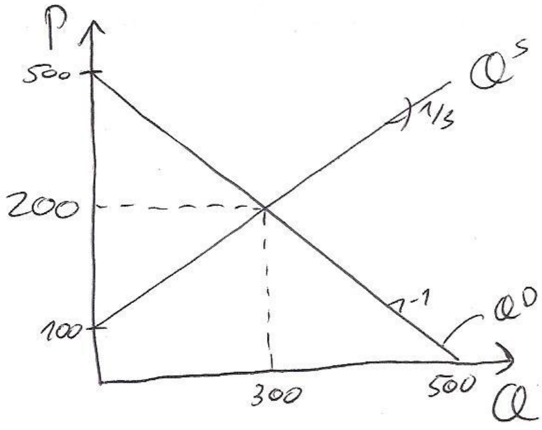
⇔ $q^s = \frac{P}{4} - 25$

Aggregate supply (n=12): $Q^s = 12q^s = \frac{12}{4}P - 25 \times 12 = 3P - 300$

$Q^s = 3P - 300$

market equilibrium: $Q^S = Q^D \Leftrightarrow 3P - 300 = 500 - P$

$\Leftrightarrow \boxed{\begin{array}{l} P = 200 \\ Q = 300 \end{array}}$

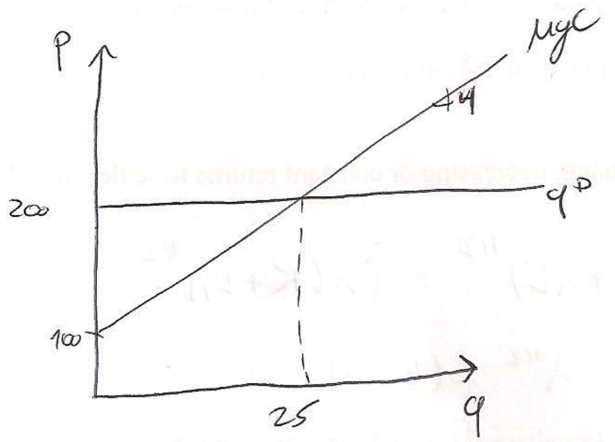


(b) How much is produced by each firm and at what price? Represent graphically the short-run equilibrium for the representative firm.

1 point

$$Q = qn \Leftrightarrow q_i = \frac{Q}{n} = \frac{300}{12} = 25$$

$$p^* = 200$$



Demand faced by each Individual firm is completely <u>ELASTIC</u> at the equilibrium price

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II
(7)

Consider a firm that operates with the following production function

$$Y = (K + L)^{1/2}$$

where labor (L) and capital (K) are the inputs needed to produce the output Y.

The prices of inputs are given by w=1 for labor and r=2 for capital.

1.

0,5 points

a) Does the firm exhibits increasing, decreasing or constant returns to scale? Why?

$y = f(K,L)$

$f(\lambda K, \lambda L) = (\lambda K + \lambda L)^{1/2} = (\lambda(K+L))^{1/2}$

$= \lambda^{1/2} (K+L)^{1/2} = \lambda^{1/2} f(K,L)$

since $\lambda^{1/2} < \lambda$ we have Decreasing returns to scale

1,5 points

b) Derive the conditional demand functions for the inputs, given the specified prices and show graphically the optimal choice of inputs.

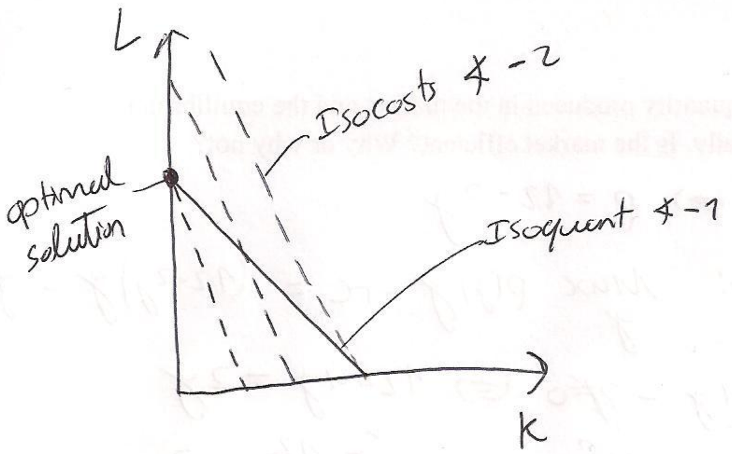
$MRTS_{KL} = \dfrac{\frac{1}{2}(K+L)^{-1/2}}{\frac{1}{2}(K+L)^{-1/2}} = 1 \Rightarrow$ There's perfect substitutability between the inputs.

since $MRTS_{KL} < \dfrac{r}{w} = \dfrac{2}{1}$, the firm will use only L (the cheapest Input)

therefore, $K^* = 0$, $L^* = y^2$ $\left(y = (\underset{\overset{\shortparallel}{0}}{K} + L)^{1/2} \Leftrightarrow y = L^{1/2} \Leftrightarrow L^* = y^2\right)$

(remember that the problem of the firm is:

$\min_{K,L} \; rK + wL$ s.to $\bar{y} = (K+L)^{1/2}$)



1,5 points

2. Suppose that the same firm as in 1. (operating with the same production function) is the only producer in the market and the market demand function is given by $Y^D = 6 - 0.5p$.

c) Determine the total cost function in the short-run for a fixed amount of capital, that is, $K = \overline{K}$.

$$K = \overline{K} \to y = (L + \overline{K})^{1/2} \Leftrightarrow \boxed{y^2 - \overline{K} = L}$$

$$TC = wL + rK$$
$$= wL + r\overline{K}$$
$$= w(y^2 - \overline{K}) + r\overline{K}$$
$$= 1(y^2 - \overline{K}) + 2\overline{K}$$
$$\boxed{TC = y^2 + \overline{K}}$$

1,5 points

d) Determine the optimal quantity produced in the market and the equilibrium price. Illustrate graphically. Is the market efficient? Why or why not?

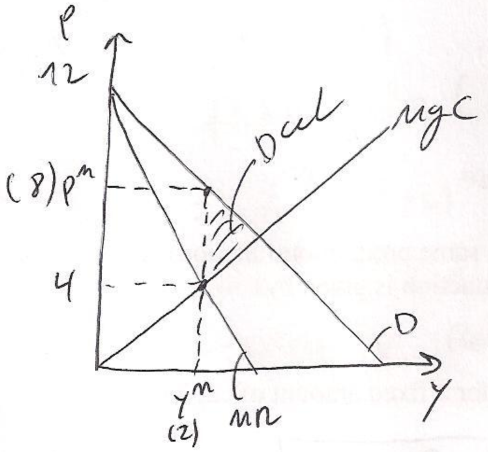
$y^D = 6 - 0{,}5p \iff p = 12 - 2y$

Firm is a monopolist: $\max_y \; p(y)y - TC = (12-2y)y - y^2 - \bar{K}$

FOC: $\frac{d\pi}{dy} = 0 \iff \underbrace{12-4y}_{MgR} - \underbrace{2y}_{MgC} = 0 \iff 12 - 4y = 2y$

$\iff y^m = \frac{12}{6} = 2$

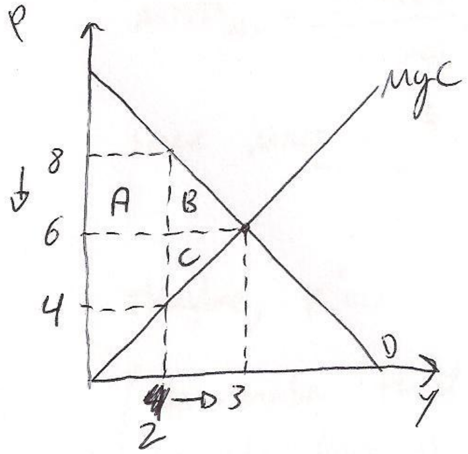
seen in Demand $p^m = 12 - 2 \times 2 = 8$



Not efficient. There's DWL since price is higher than marginal cost

1 point

e) Suppose that the government sets the price at 6. Compute the change in consumer surplus, profits and total welfare with respect to question d). Illustrate these changes graphically.



$\Delta CS = A + B = (8-6) \times 2 + \frac{(8-6) \times 1}{2} =$

$= 4 + 1 = 5$

$\Delta \pi = -A + C = -(8-6) \times 2 + \frac{(6-4) \times 1}{2}$

$= -4 + 1 = -3$

$\Delta W = (A+B) + (-A+C) = B + C$

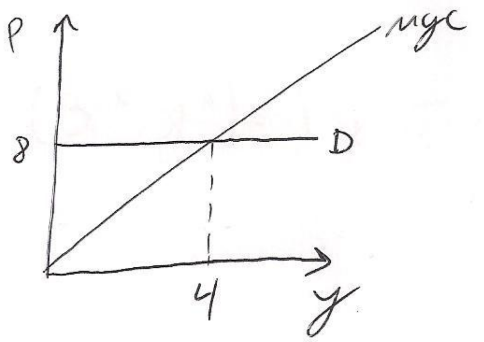
$= 2$

0,5 points

f) Suppose that the demand is now perfectly elastic at $p^D = 8$. Derive the new equilibrium quantity and price. Is the market efficient? Why or why not?

$$\max_{y} \; Py - TC = 8y - y^2 - \bar{K}$$

$$\text{FOC: } 8 = 2y \Leftrightarrow \boxed{\begin{array}{l} y^* = 4 \\ p^* = 8 \end{array}}$$



market is efficient since welfare is maximized when $p = mgc$ (no DWL)

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0,5 points

g) How do you relate the equilibrium in f) with a perfect competitively equilibrium? Explain briefly.

Since demand is completely elastic, the firm cannot choose another price than 8.

In this case, the ~~Mgr~~ marginal revenue will be equal to the price so the solution will be $MgR = P = MgC$. We have the same solution as in perfect competition!

A monopoly with a perfectly elastic demand is efficient.

III
(7)

Consider a consumer with the following utility function:

$$U(x, y) = xy$$

Let $p_x = 10$ and $p_y = 2$ represent the unitary prices of good x and y, respectively. Assume that the income of the consumer is $M = 100$.

1,5 points.

a) Derive the demand functions for x and y and determine the optimal choice given the specified prices and income.

$$\max_{x,y} \; U = xy \quad \text{s.to} \quad P_x x + P_y y = M$$

$$\begin{cases} |MRS| = \frac{y}{x} = \frac{P_x}{P_y} \\ P_x x + P_y y = M \end{cases} \Leftrightarrow \begin{cases} \frac{y}{x} = \frac{P_x}{P_y} \\ \text{—} \end{cases} \Leftrightarrow \begin{cases} y = x \frac{P_x}{P_y} \\ P_x x + \cancel{P_y}\, x \frac{P_x}{\cancel{P_y}} = M \end{cases}$$

$$\Leftrightarrow \begin{cases} y = \frac{M}{2\cancel{P_x}} \frac{\cancel{P_x}}{P_y} \\ x = \frac{M}{2P_x} \end{cases} \Leftrightarrow \begin{cases} y^* = \frac{M}{2P_y} \\ x^* = \frac{M}{2P_x} \end{cases}$$ Demand functions

$$(x^*, y^*) = \left(\frac{100}{2\times 10} ; \frac{100}{2\times 2}\right) = (5, 25)$$

1 point

b) Suppose your income increased by 20. What is the new optimal choice? Illustrate graphically.

$$\frac{dy^*}{dM} = \frac{1}{2P_y} \Leftrightarrow dy^* = \frac{dM}{2P_y} = \frac{20}{4} = 5 \rightarrow y'^* = 30$$

$$\frac{dx^*}{dM} = \frac{1}{2P_x} \Leftrightarrow dx^* = \frac{dM}{2P_x} = \frac{20}{2\times 10} = 1 \rightarrow x'^* = 6$$

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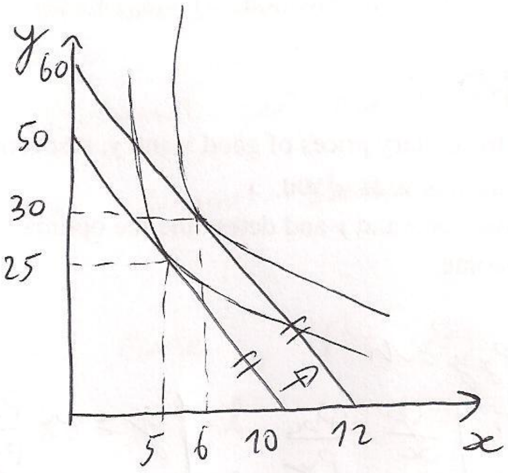


0,5 points

c) Compare the results obtained in b) with those that you would have obtained with a quasilinear utility function.

Only the good with a linear impact on utility would have its consumption Increased.
The good with a non-linear impact on utility has no Income effect.

d) Suppose the price of y increases to 5. How much would you be willing to pay to avoid the price increase? *(Note: You do not have to present the simplified value, an expression is enough)*

1,5 points

Hicksean Equivalent variation

- Final bundle: $(x^*, y^*) = \left(5, \frac{100}{2\times5}\right) \to (5, 10)$
- Final utility $V_F = 10 \times 5 = 50$

Required M' to obtain the final utility at the initial prices:

$$50 = \left(\frac{M'}{2\times2}\right)\left(\frac{M'}{2\times10}\right) \Leftrightarrow M' = \sqrt{4000} \approx 63$$

$$\Delta M = \sqrt{4000} - 100 = 63 - 100 = -37 = WTP$$

1,5 points

e) And what is the minimum compensation you would be willing to accept for the price increase? *(Note: You do not have to present the simplified value, an expression is enough)*

Hicksean compensating variation

- Initial bundle: $(x^*, y^*) = (5, 25)$
- Initial utility: $V_I = 5 \times 25 = 125$

Required M' to obtain initial utility at the final prices:

$$125 = \left(\frac{M'}{2\times5}\right)\left(\frac{M'}{2\times10}\right) \Leftrightarrow M' = \sqrt{25000} \approx 158$$

$$\Delta M = 158 - 100 = 58 = WTA$$

0,5 points

f) Given the results obtained in the previous two questions, what can you say about the value of the change in consumer surplus?

Since we have a normal good and the price increased:

$|CV| \geq \Delta CS \geq |EV|$

$58 \geq \Delta CS \geq 37$

g) If you had a quasilinear utility function, with y being the good with a non-linear impact in utility, how would you relate questions d), e) and f)? Explain briefly.

0,5 points

Since there's no Income effect:

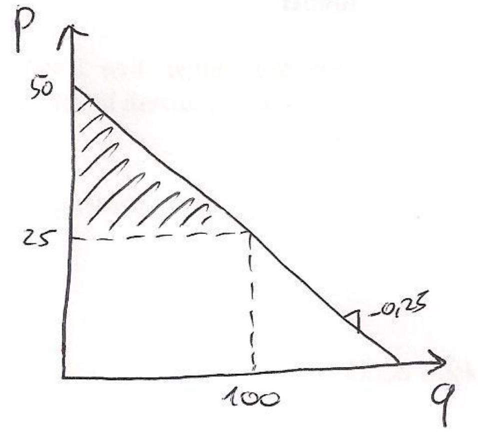
$|CV| = \Delta CS = |EV|$

IV
(3)

Imagine that a tennis club rents its courts for 25 € per person per hour. Assuming that Maria's demand curve for court time is given by

$$p = 50 - 0.25q$$

where q is measured in hours per year, and that there are no other tennis courts close by, what is the maximum annual membership fee that Maria would be willing to pay for the right to buy court time for 25€ per hour?



$p = 25$

$\Rightarrow 25 = 50 - 0{,}25q$

$\Leftrightarrow q = \dfrac{25}{0{,}25} = 100$

$$CS = (50-25)\frac{100}{2} = 1250$$

The maximum she's willing to pay is 1250. If the fee is higher than this, the surplus she gets from playing tennis is negative so she wouldn't play.