Problem Set VI Solution

Prepared by the Teaching Assistants

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1. Question 1

   We are in the short-run and assume that P_1 is fixed by the firms. They commit to supplying whichever amount of output is being demand-determined. It so happened that this price level is the one that hits the long-run equilibrium, as such it is assumed that both the goods and labour market clear with P_1.
   We now assume that the Central Bank decides to lower the interest target r_1 to r_2.
   In the goods market, a decrease in the interest rate, increases the demand for output (as it increases both consumption and investment), we move along the Y^D curve. Y_2 will now be our new output level.
   In the money market, an increase in Y and a decrease in r increases M^D. The money market will not be in equilibrium, given P_1, unless the Central Bank increases money supply, as such M^S increases to M_2.
   In the labour market, because the interest rate decreased, the N^S will move to the left. Now, the firms will hire just enough to make sure that Y_2 is produced. They will hire more labour, but to do so they have to pay the efficient wage w_2, so that the workers would be willing to supply N_2 (the amount of labour units that supports Y_2).
   In the end: $\uparrow Y$, $\uparrow C \uparrow I \uparrow N \uparrow w$. Check the box on page 543, 5^{th} edition, to see how these predictions match the data.

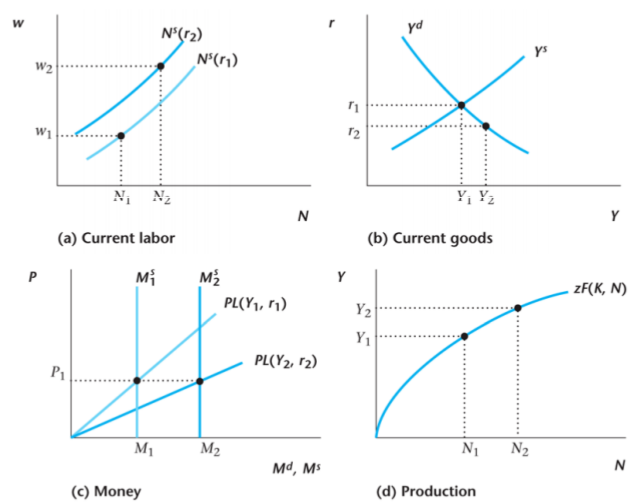

Figure 1: NKM decrease in r

2. Question 3

b) 

c) We obtain recessions and expansions from the shocks generated in the economy. The cycles and its durations depend on the process from where the shocks are generated and the persistence of it. Consumption and investment vary very close to the output, in fact the correlation between them and output is 1. This is not surprising given the functional forms of both variables as a function of k_t and z_t. Indeed both can be written as a linear function of y_t. The interest rate nevertheless is more fluctuating, and is because it adjusts itself depending on the shock, recall that in this model is a price that allows the reallocation of resources over time (savings, investment or capital). As labor is fixed, wages are perfectly correlated with the output.
The table of correlations is:

	y_t
c_t	1
i_t	1
r_t	0.18
w_t	1

Table 1: Correlations for Question 3.c

3. Question 4.

(a)

$$\max_{c_t, c_{t+1}} \log(c_t^t) + \log(c_{t+1}^t) \tag{1}$$

$$s.t. \quad c_t^t + k_{t+1} = w_t \tag{2}$$

$$c_{t+1}^t = r_{t+1}k_{t+1} + (1-\delta)k_{t+1} \tag{3}$$

Replacing k_{t+1} from 2 into 3 we obtain:

$$\begin{aligned} c_{t+1}^t &= r_{t+1}(w_t - c_t^t) + (1-\delta)(w_t - c_t^t) \\ c_t^t[(1-\delta) + r_{t+1}] + c_{t+1}^t &= [r_{t+1} + (1-\delta)]w_t \\ c_t^t + \frac{c_{t+1}^t}{(1-\delta) + r_{t+1}} &= w_t \end{aligned}$$

As the consolidated budget constraint. Note that k_{t+1} would be like “savings” in the previous model, which is actually what it is. Now let us state the maximization problem again:

$$\max_{c_t, c_{t+1}} \log(c_t^t) + \log(c_{t+1}^t)$$

$$s.t. \quad c_t^t + \frac{c_{t+1}^t}{(1-\delta) + r_{t+1}} = w_t$$

The FOC with respect to c_{t+1}^t lead us to

$$\frac{1}{c_t^t}\frac{\partial c_t^t}{\partial c_{t+1}^t} + \frac{1}{c_{t+1}^t} = 0$$

, and recalling that from the Budget Constraint we can obtain

$$\frac{\partial c_t^t}{\partial c_{t+1}^t} = \frac{-1}{(1-\delta) + r_{t+1}}$$

the FOC would finally be:

$$\frac{1}{c_t^t}\frac{-1}{(1-\delta) + r_{t+1}} + \frac{1}{c_{t+1}^t} = 0$$

$$\frac{c_{t+1}^t}{c_t^t} = (1-\delta) + r_{t+1} \tag{4}$$

Using 4 into the budget constraint, we can find c_t as a function of w_t, later we will use that to obtain k_{t+1} from 2.

$$\begin{aligned}
c_t^t + c_t^t &= w_t \\
c_t^t &= \frac{w_t}{2} \\
c_{t+1}^t &= \frac{w_t[(1-\delta) + r_{t+1}]}{2} \\
k_{t+1} &= w_t - c_t = w_t - \frac{w_t}{2} = \frac{w_t}{2}
\end{aligned}$$

(b) The Problem of the Firm, is to maximize profits, so:

$$\begin{aligned}
&\max_{N_t, k_t} z_t k_t^\alpha N^{1-\alpha} - w_t N_t - r_t k_t \\
FOC_N \quad &(1-\alpha) z_t k_t^\alpha N_t^{-\alpha} = w_t \\
FOC_k \quad &\alpha z_t k_t^{\alpha-1} N_t^{1-\alpha} = r_t
\end{aligned}$$

Now imposing $z_t = 1$ and $N_t = 1$:

$$\begin{aligned}
w_t &= (1-\alpha) k_t^\alpha \\
k_{t+1} &= \frac{(1-\alpha) k_t^\alpha}{2}
\end{aligned}$$

Imposing steady state, $k_{t+1} = k_t = k_{ss}$:

$$k_{ss} = \frac{(1-\alpha)k_{ss}^{\alpha}}{2}$$
$$k_{ss}^{1-\alpha} = \frac{(1-\alpha)}{2}$$
$$k_{ss} = \left(\frac{1-\alpha}{2}\right)^{\frac{1}{1-\alpha}}$$

Now, with that, we can solve for all our variables that where function of k_t, now in the steady state, and imposing $z_t = 1$ and $N_t = 1$:

$$r_{ss} = \alpha \left(\frac{1-\alpha}{2}\right)^{\frac{\alpha-1}{1-\alpha}} = \frac{2\alpha}{1-\alpha}$$
$$w_{ss} = (1-\alpha)\left(\frac{1-\alpha}{2}\right)^{\frac{\alpha}{1-\alpha}} = 2\frac{(1-\alpha)}{2}\left(\frac{1-\alpha}{2}\right)^{\frac{\alpha}{1-\alpha}} = 2\left(\frac{1-\alpha}{2}\right)^{1+\frac{\alpha}{1-\alpha}} = 2\left(\frac{1-\alpha}{2}\right)^{\frac{1}{1-\alpha}}$$
$$c_{ss} = \left(\frac{1-\alpha}{2}\right)^{\frac{1}{1-\alpha}}$$

With those variables, is easy to find by replacement y_{ss} and i_{ss}.

c) First let's make the graphs:

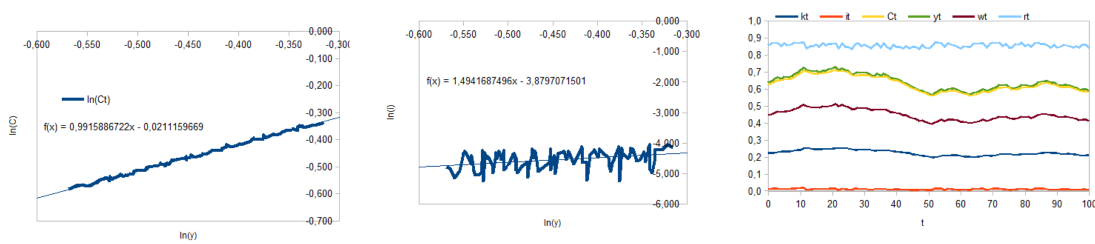

Table 2: For $\delta = 0.05$

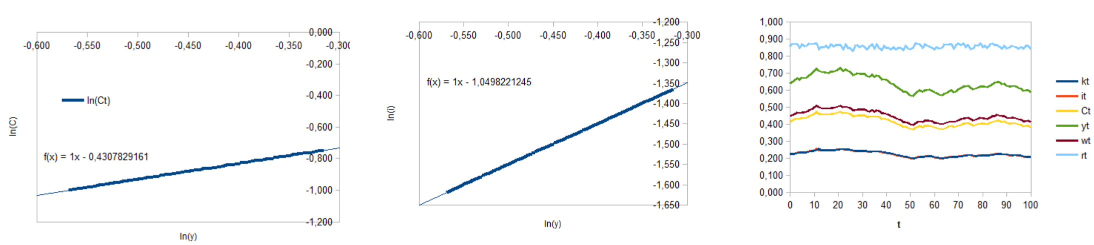

Table 3: For $\delta = 1$

We can observe that when we have full depreciation consumption is almost equal to the wage. This makes sense because the Output has to go to capital formation in a bigger share, before, almost all output in steady state went to consumption and just a little bit of it to keep the capital stock in the steady state level. Note in the same direction, that investment in the $\delta = 0.05$ case, investment is almost zero, instead, in the $\delta = 1$ case is attached to k_t. That same patter is observed in the trend of the log-series with respect to output.

d) The evolution of the variables for a permanent shock is:

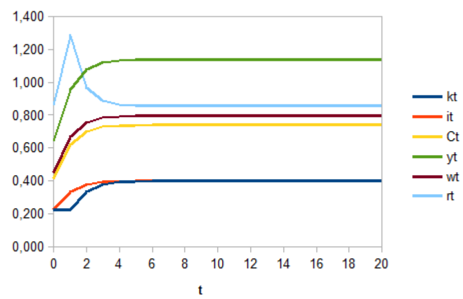

Figure 2: With a permanent shock

With a permanent shock, investment jumps to move the stock of capital to the new steady state value, and then vanish to be enough to cover depreciation. The interest rate jumps up to the new level because of the jump on the marginal product of capital, but then decreases with the new investment, increasing stock of capital and decreasing as well the marginal productivity, but to a higher level.

e) The evolution of the variables for a temporary shock is:

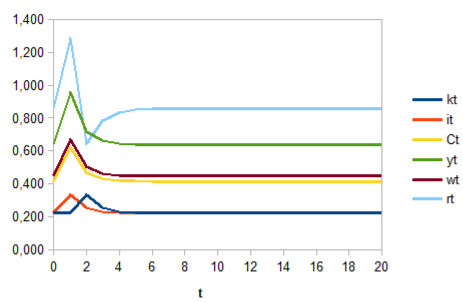

Figure 3: With a temporary shock

Now the shock is temporal, so we observe the jumps but then go back to the original levels. The interest rate jumps because of the increasing in the marginal productivity of capital, as well the investment, then they come back to the original levels, but the investment becomes negative and consumption positive, to push the stock of capital to the original optimal level.