Nova School of Business and Economics
Macroeconomics 1103, 2012-2013, 1st Semester
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Problem Set 6
TA Solution

1.

The main equations of this model are

$$\begin{aligned} y_t &= z_t k_t^{\alpha} \\ w_t &= (1-\alpha)\, y_t \\ r_t &= \alpha \frac{y_t}{k_t} \\ k_{t+1} &= i_t = \alpha\beta y_t \\ c_t &= (1-\alpha\beta)\, y_t = y_t - i_t \\ z_t &= z_{t-1}^{0.95} \left(\exp \varepsilon_t\right)^{0.05},\ \epsilon_t \sim Uniform\left([-0,5,0,5]\right). \end{aligned}$$

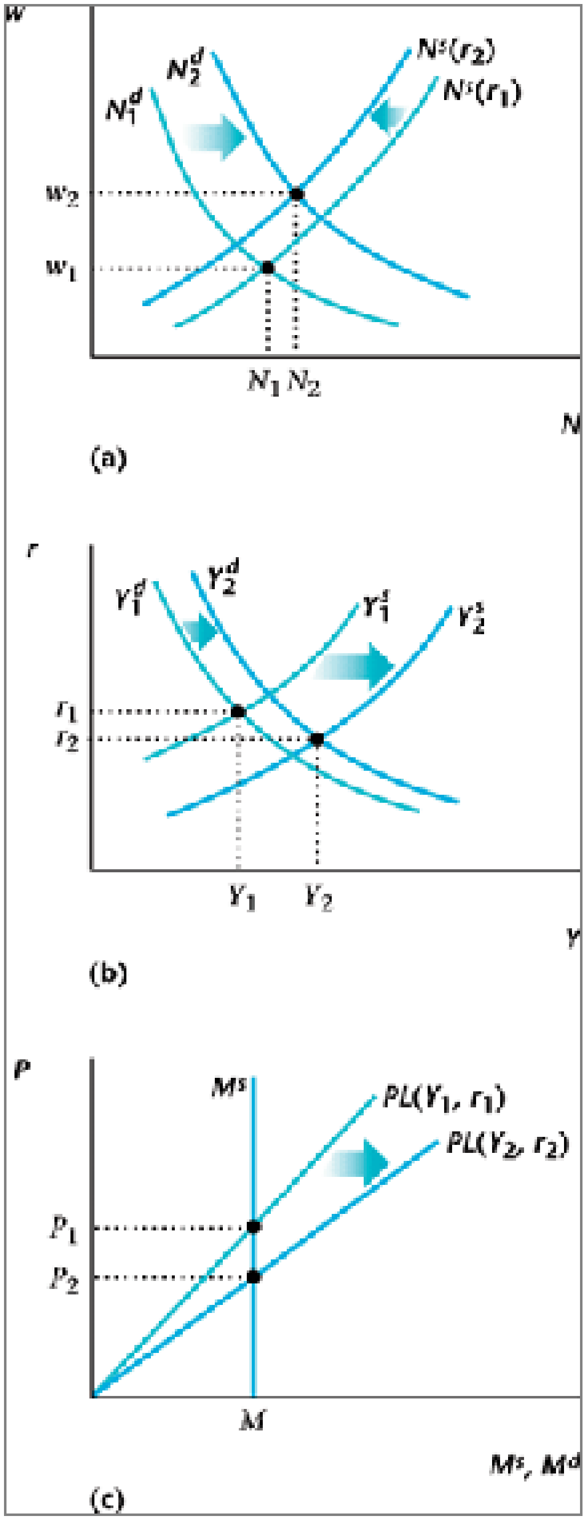
The parameters of the model are given to us as follows,

$$\begin{aligned} z_0 &= 1 \Rightarrow \varepsilon_0 = 0 \\ \alpha &= 0.3 \\ \beta &= \frac{1}{1.05} \\ k_0 &= (\alpha\beta)^{\frac{1}{1-\alpha}}. \end{aligned}$$

The answers to the first exercise can be found in the auxiliary excel file.

2.

a)

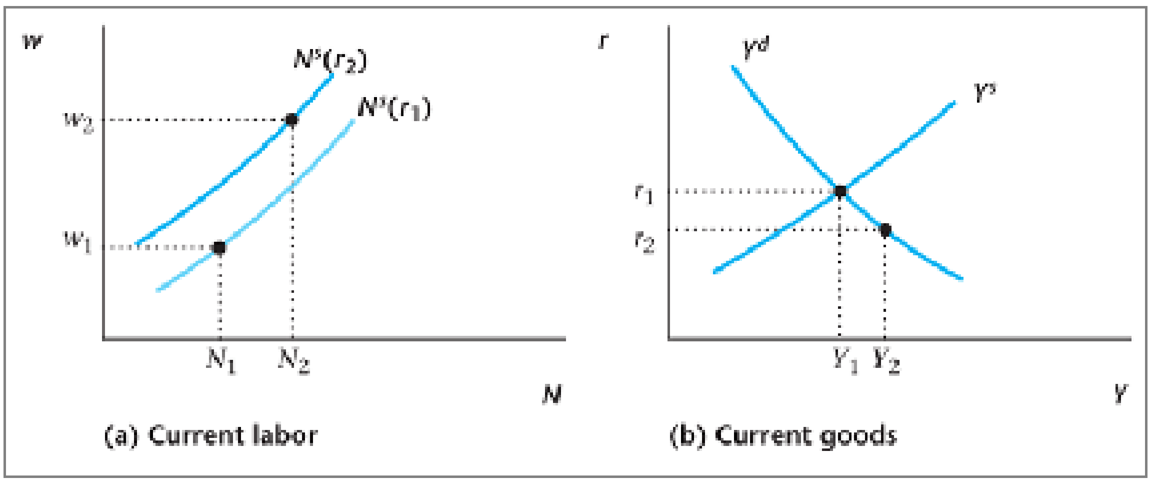


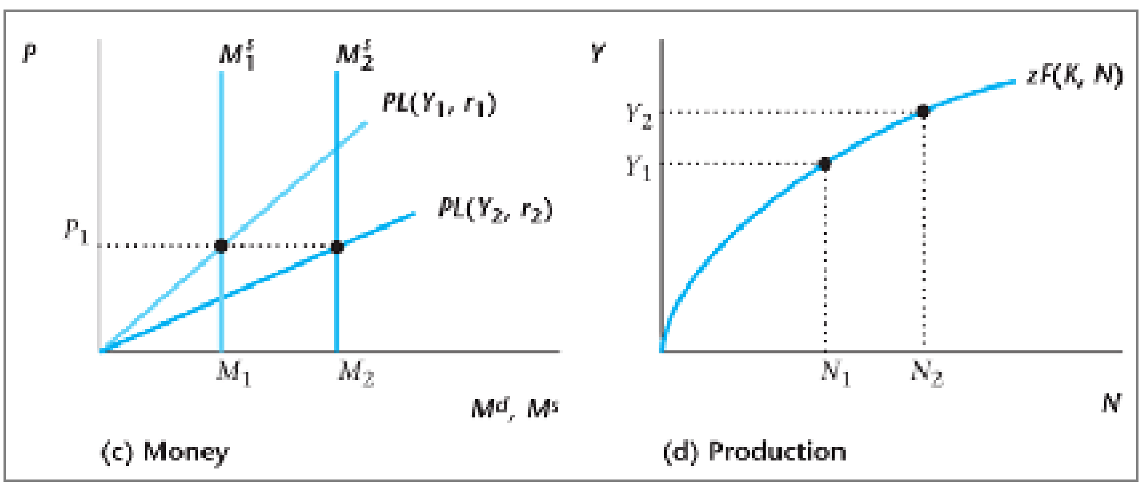
w
N_2^d
N_1^d
$N^s(r_2)$
$N^s(r_1)$
w_2
w_1
N_1 N_2
N
(a)
r
Y_1^d
Y_2^d
Y_1^s
Y_2^s
r_1
r_2
Y_1
Y_2
Y
(b)
P
M^s
$PL(Y_1, r_1)$
$PL(Y_2, r_2)$
P_1
P_2
M
M^s, M^d
(c)

$$\downarrow P, \uparrow w, \uparrow N, \downarrow r, \uparrow Y$$

See pages 441-445 of the text-book.

b)





$$\uparrow w, \uparrow N, \uparrow Y$$

See pages 480-481 of the text book.

3.

a) Solving the budget contraints of each period in order to c_t^t and c_{t+1}^t and replacing on the objective function we obtain,

$$\max_{k_{t+1}} \ln\left(w_t - k_{t+1}\right) + \ln\left(r_{t+1}k_{t+1} + (1-\delta)\,k_{t+1}\right).$$

Taking the first order condition we reach the optimal decision of k_{t+1} in order to the wage,

$$1 : \frac{-1}{w_t - k_{t+1}} + \frac{r_{t+1} + (1-\delta)}{\left(r_{t+1} + (1-\delta)\right)k_{t+1}} = 0 \Leftrightarrow \frac{1}{k_{t+1}} = \frac{1}{w_t - k_{t+1}} \Leftrightarrow k_{t+1} = w_t - k_{t+1} \Leftrightarrow k_{t+1} = \frac{w_t}{2}.$$

b) At the optimum $w_t = MP_{N_t}$ and $r_t = MP_{k_t}$. In this way for a production function, $y_t = z_t k_t^{\alpha} N_t^{1-\alpha}$, we have that

$$\begin{aligned} w_t &= \frac{\partial y_t}{\partial N_t} = (1-\alpha)\,z_t\left(\frac{k_t}{N_t}\right)^{\alpha} \\ r_t &= \frac{\partial y_t}{\partial k_t} = \alpha z_t\left(\frac{k_t}{N_t}\right)^{\alpha-1}. \end{aligned}$$

Replacing for $N_t = z_t = 1$ we get

$$\begin{aligned} 2 &: \ w_t = (1-\alpha)\,k_t^{\alpha} \\ 3 &: \ r_t = \alpha k_t^{\alpha-1}. \end{aligned}$$

Replacing 2 in 1 we obtain

$$k_{t+1} = \frac{w_t}{2} \Leftrightarrow k_{t+1} = \frac{(1-\alpha)}{2}k_t^{\alpha}.$$

Knowing that in the steady state capital is constant $(k_{t+1} = k_t = k)$ we achieve its equilibrium value

$$k = \left(\frac{1-\alpha}{2}\right)^{\frac{1}{1-\alpha}}.$$

Replacing k in equations 2 and 3 we can derive the steady state values of the real wage and interest rate,

$$\begin{aligned} w &= (1-\alpha)\left(\frac{1-\alpha}{2}\right)^{\frac{\alpha}{1-\alpha}} = \left(\frac{1-\alpha}{2^{\alpha}}\right)^{\frac{1}{1-\alpha}} \\ r &= \alpha\left(\frac{1-\alpha}{2}\right)^{-1} = \frac{2\alpha}{1-\alpha}. \end{aligned}$$

c) See excel file.
d) See excel file.
e) See excel file.