Name: ____. Code: ______________

Nova School of Business and Economics
Macroeconomics 1103, 2012-2013, 1st Semester
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TA: João Morgado

Problem Set 6

Due Date: Thursday, November 29, at the beginning of the class
Once I start the class, I will not accept problem sets.

Turning in the problem sets is optional. For those that decide to turn in the problem sets, they have to be turned in on paper. The problem sets can be done in groups, but they have to be turned in individually.

Please, turn in your problem set with this cover page, with your name and code filled out above.

1. Business cycle simulation. Continue working from question 4 of the last problem set to do your own business cycle simulation.

From the last problem set, the production function is $y_t = z_t k_t^\alpha N_t^{1-\alpha}$. Suppose that z_t varies according to $z_t = z_{t-1}^{0.95} e^{0.05\varepsilon_t}$, where ε_t is a random shock with uniform distribution $[-0.5, +0.5]$. ε_t are the productivity shocks. Suppose that $z_0 = 1$, $\beta = \dfrac{1}{1.05}$ and $\alpha = 0.3$. Consider that the economy starts at the steady state. That is, k_0 is equal to the value of capital such that $k_{t+1} = k_t$, calculated on item b from the last problem set.

Using Excel, define columns for t, ε_t, z_t, i_t, c_t, y_t, w_t, and r_t. Make a simulation with 100 periods, $t = 0, 1, ..., 100$. Use the function $rand()$ in Excel to define a value for ε_t for each period. To do this, write $= rand() - 0.5$ in the cells. To avoid having the values of ε constantly changing, use "paste especial," "values", and copy and paste the values of the column over itself.

c. With $z_0 = 1$, obtain the values of z_t for $t = 1, 2, ...$ With k_t and z_t, calculate i_t, c_t, y_t, w_t, and r_t.

d. Make a graph of i_t, c_t, y_t over time. Make a graph of w_t, r_t, c_t, and i_t versus y_t (use "scatter plot," do not connect the points).

e. Discuss your results. Do you obtain expansions and recessions? Do you obtain cycles with variable duration? How do consumption and investment vary with output? What about the interest rate? What is the correlation between these variables and output?

2. Obtain the predictions of the models below about GDP, consumption, wages, and other macroeconomic variables after the shocks indicated. Consider that the economy is initially in equilibrium. Use diagrams to answer. Compare the predictions with the data.

a. Persistent shocks on total factor productivity, in accordance to the Real Business Cycles model (ch. 12, pp. 441-445).

b. Shocks to the interest rate, in accordance to the New-Keynesian model (ch. 13, p. 482).

3. Question 1 simplifies the calculations by having full depreciation, $\delta = 1$. However, capital varies strongly because of this assumption. We will now work with an economy with $\delta < 1$.

The solution obtained in question 1 cannot be applied with $\delta < 1$. So, we will simplify the economy in a different way. It is still possible to obtain a solution for question 1 with $\delta < 1$, but we would have to use numerical methods instead of obtaining the solution analytically, as we do here.

We will use overlapping generations. This method is used in many applications, especially for the analysis of social security. This exercise, with some modifications

on notation, is taken from chapter 9 of DLS. There is a link to this book on the course webpage.

With overlapping generations, a new generation is born in every period. Let us say that each generation lives for two periods. In each period, there is a young and an old generation. The young were born in the same period. The old were born one period before. The optimization problem of a person that was born in period t is

$$\begin{aligned}&\max \ \ln c_t^t + \ln c_{t+1}^t \text{ s.t.}\\ &c_t^t + k_{t+1} = w_t\\ &c_{t+1}^t = r_{t+1} k_{t+1} + (1-\delta)\, k_{t+1}.\end{aligned}$$

In the first period, the consumer works, receives w_t as labor income, and saves k_{t+1}. In the second period, as old, the consumer lends the capital k_{t+1} to production at the beginning of the period and receives interest payments plus depreciated capital at the end of the period. The consumer works $N_t = 1$.

a. Obtain the optimal decision of k_{t+1} as a function of the wage.

b. The production function is given by $y_t = z_t k_t^{\alpha} N_t^{1-\alpha}$. Write the value of wages w_t and interest rates r_t as a function of k_t.

With $N_1 = 1$ and your answer in item *a*, obtain capital, wages, and interest rates in the steady state (with $z_t = 1$ for all periods).

Aggregate consumption at t is given by the sum of the consumption of the young and of the old, $C_t = c_t^t + c_t^{t-1}$. Aggregate consumption plus investment is equal to total production,

$$c_t^t + c_t^{t-1} + i_t = z_t k_t^{\alpha} N_t^{1-\alpha} = y_t.$$

Investment is given by $i_t = k_{t+1} - (1-\delta)\, k_t$. k_t is equal to capital at t and k_{t+1} is equal to your solution in item *a*.

Suppose that $\alpha = 0.3$ and $\delta = 0.05$. Also, suppose that z_t varies as in question 1, $z_t = z_{t-1}^{0.95} e^{0.05\varepsilon_t}$, with ε_t uniform $[-0.5, 0.5]$ and $z_0 = 1$. Create columns for z_t, y_t, i_t, k_t, C_t, w_t and r_t in Excel and analyze the evolution of the variables. Capital at $t = 0$ is equal to capital in the steady state.

c. Does the evolution of the variables agree with the data on economic fluctuations? With a scatter plot, calculate the coefficient b in $\log C_t = a + b \log y_t$. To do this, it is enough to add a trend and select the option to show the equation. Do the same for $\log i_t$. What do you obtain? How do your results change with full depreciation, $\delta = 1$?

d. Permanent shock. Suppose that $z_0 = 1$ and that $z_t = 1.5$ for all $t > 0$. Show the evolution of the variables over time. What happens to investment? And to the interest rate? How to explain the evolution of r_t and i_t?

e. Temporary shock. Do the same for $z_0 = 1$, $z_1 = 1.5$, and $z_t = 1$ for $t \geq 2$.