Problem Set V Solution

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1. Question 1

   Yes, the behaviour described by TSF is in accordance with the consumption smoothing hypothesis which points out that consumption is much less volatile than income. This behavior implies that savings will decrease (or even become negative during Christmas season) as the Portuguese consumers "redistribute" their "future income" to the present to avoid a big variation in their consumption patterns.

2. Question 2

   This question is placed in the framework of the monetary intertemporal model in which we have seen the money neutrality result. The Friedman-Lucas money surprise model adds some rigidities to our model by assuming that the consumers/workers have imperfect information regarding aggregate variables. They know their nominal wage W, but have imperfect information about the price level P and consequently about $w = \frac{W}{P}$ - the real wage . They are also unaware of the source of shocks. In our simple experiment, we assume that a shock may have derived either by a temporary increase in z or a permanent increase in M^S. We have studied them separately:

   - $\uparrow z$ increases the real wage rate w and decreases the price level P;
   - $\uparrow M^S$ leads to a increase in P and the real wage w is left unchanged, that is, $\Delta W = \Delta P$.

   Workers care about the real wage. Under perfect information when $\uparrow M^S$ they will not change labour supply, hence Y unchanged. But when $\uparrow z$ they know that $\uparrow w$ and so they increase labour supply and $\uparrow Y$.

   We suppose that the monetary authority increased M^S but agents have imperfect information. We know that this causes the nominal wage W to increase. Workers may perceive this as having been $\uparrow z$ that causes $\uparrow w$. They will, therefore, increase labour supply. The equilibrium dynamics tell us that $\uparrow N, \downarrow w, \uparrow Y, \downarrow r$ and $\uparrow \frac{M}{P}, \Delta M > \Delta P$.

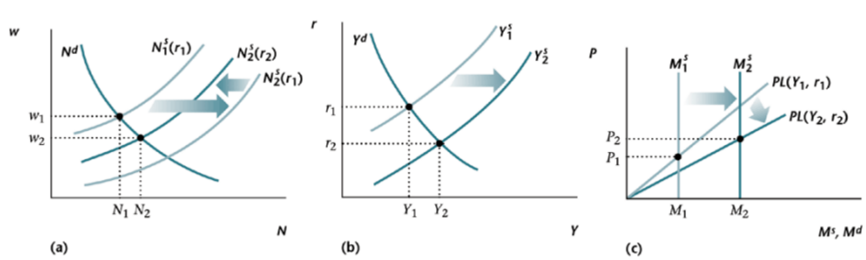

Figure 1: Money Surprise Shock

   We conclude that in this setting money is not neutral. This tells us that the government may use *unantecipated* increases in money supply to confuse imperfectly informed (though still rational)

agents to work harder. However, this money surprise only works in the short-run: you may fool people once, but not twice or thrice. The natural policy implication is the so called Friedman's rule that states that there should be a constant growth of money supply.

3. Question 3

   The RBC Theory points the shocks on the productivity today and tomorrow as the main drivers behind the generation of business cycles in the economy.

   This is the case to explore in this question where the technology, z, changes today, but also in a future period, z'. The graphical impact of these shocks can be depicted as:

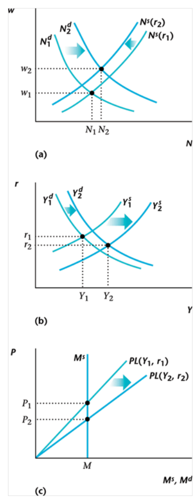

Figure 2: Persistent technological schock

   Dividing the analysis in the two shocks we have:

   The shock in z:

   1. Since a better technology today increases the marginal productivity of labour that leads to an upward shifting in the labour demand, increasing the hours of work in the economy.

   2. Since the production function has a positive first derivative in relation to N, the higher hours of work in the economy will then lead to an expansion of the labour Supply, from Y_1^s to Y_2^s

   The shock in z':

   1. The increase in z' will be responsible be responsible for a higher level of production tomorrow. This means that every investment made today has now a higher marginal benefit in the future, and consequently the investment increases . Adding to this, it must be added the anticipation of higher income in the future, which is responsible for consumption to go up. The two previous facts make the demand expand from Y_1^d to Y_2^d. On the aggregate the output increases, however the interest rate must decrease, since the impact on the technology today is expected to be higher than the technological improvement in the future.

   2. Given the falling of the interest rate, the supply of Labour will shrink, from N_1^s to N_2^s.

   On the aggregate, the model predicts the following impacts over the variables:

   Output, hours of work wages and consumption increase

   Interest rate and prices decrease.

4. Question 4

Since $N_t = 1$ in equilibrium we may express y_t in terms of (z_t, k_t) as follows:

$$y_t = z_t k_t^{\alpha}.$$

Regarding w_t and r_t we can also express them in terms of (z_t, k_t) through the solution to the representative firm's problem,

$$\begin{aligned} w_t &= (1-\alpha) z_t k_t^{\alpha} \\ r_t &= \alpha z_t k_t^{\alpha-1}. \end{aligned}$$

From the consumer's problem solution we already know that

$$k_{t+1} = \beta \alpha z_t k_t^{\alpha}.$$

Since $\delta = 1$, $i_t = k_{t+1}$, investment will have the same expression,

$$i_t = k_{t+1} = \beta \alpha z_t k_t^{\alpha}.$$

Finally, from the consumer's budget constraint we have that

$$\begin{aligned} c_t + k_{t+1} &= w_t + r_t k_t \\ c_t &= w_t + r_t k_t - k_{t+1}. \end{aligned}$$

Replacing k_{t+1}, w_t and r_t we obtain

$$\begin{aligned} c_t &= (1-\alpha) z_t k_t^{\alpha} + \alpha z_t k_t^{\alpha-1} k_t - \beta \alpha z_t k_t^{\alpha} \\ &= (1-\alpha) z_t k_t^{\alpha} + \alpha z_t k_t^{\alpha} - \beta \alpha z_t k_t^{\alpha} \\ &= z_t k_t^{\alpha} - \beta \alpha z_t k_t^{\alpha} \\ &= (1-\beta\alpha) z_t k_t^{\alpha} = (1-\beta\alpha) y_t. \end{aligned}$$

b) The solution to the consumer's problem relates k_{t+1} to k_t as follows,

$$k_{t+1} = \beta \alpha z_t k_t^{\alpha}.$$

Replacing $z_t = 1$ we obtain

$$k_{t+1} = \beta \alpha k_t^{\alpha},$$

the equation that describes the evolution of k_t for $z_t = 1$ for all t.

The value of k_t such that $k_t = k_{t+1}$ is the so called steady state value. Once capital reaches the steady state value, it remains constant at that point.

In this way for $k_t = k_{t-1} = k_{SS}$,

$$
\begin{aligned}
k_{SS} &= \beta\alpha k_{SS}^{\alpha} \\
k_{SS}^{1-\alpha} &= \beta\alpha \\
k_{SS} &= (\beta\alpha)^{1/(1-\alpha)}
\end{aligned}
$$

At the steady state capital is a function of β and α.

If k_0 is equal to k_{SS}, capital will be fixed. In this way, output, consumption, investment, the real wage and the real interest rate will be fixed as well.