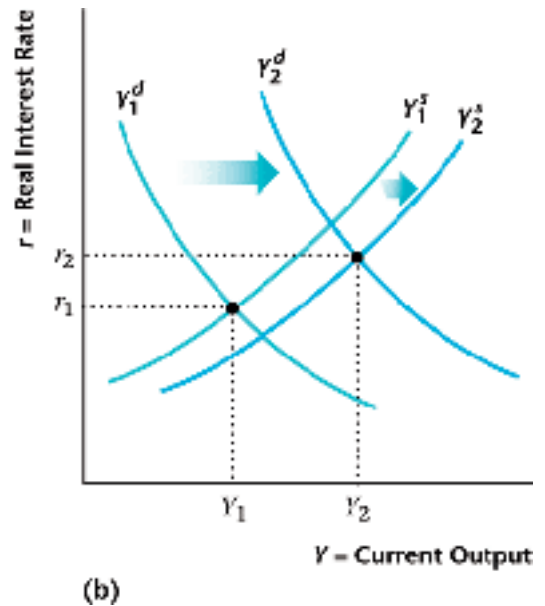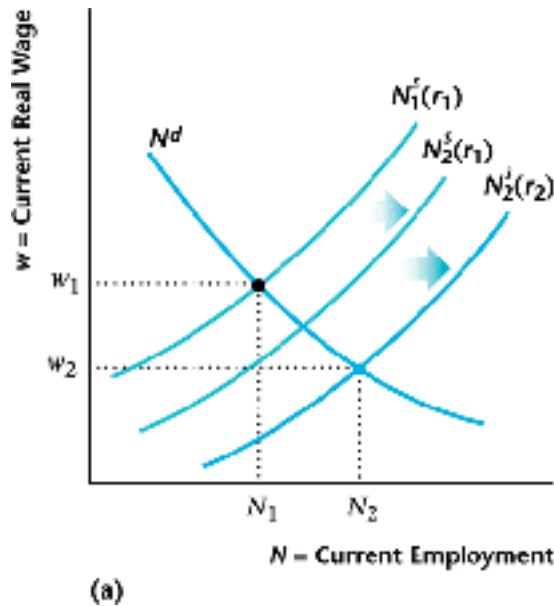Problem Set 5
TA Solution

1.

Yes, the behavior described by TSF is in accordance with the consumption smoothing hypothesis which points out that consumption is much less volatile than income. This behavior implies that savings will decrease (or even become negative during Christmas season) as the Portuguese consumers "redistribute" their "future income" to the present to avoid a big variation in their consumption patterns.
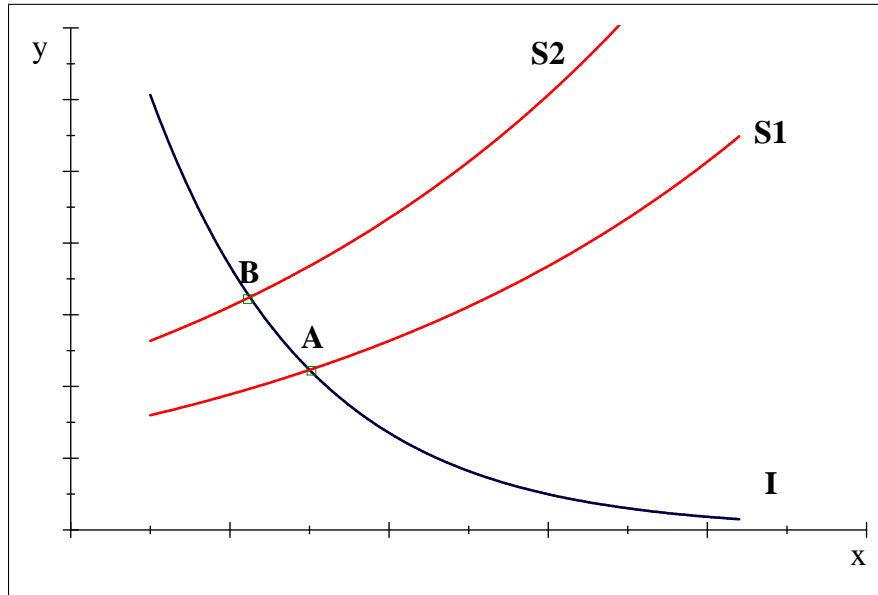
2.
a)

(a)

(b)

$$\downarrow w, \uparrow N, \uparrow r, \uparrow Y$$

As the figure depicts an increase in G leads to a horizontal shift to the right on the output demand curve equal to the variation in government spending given that the partial expenditure multiplier (ratio of the total increase in demand for goods over the increase in government spending) is one in this model. Regarding

the supply curve it will shift to the right as well, since workers will be willing to offer more labor for any given interest rate due to the additional taxes implied by the increase in G (recall the Ricardian Equivalence). Assuming that the increase in G in only temporary, this will have a small impact on the lifetime wealth of the consumer, which in turn will produce a small effect on the labor supply curve for the initial interest rate. Therefore, the impact on the output supply curve will be smaller than the impact on the output demand curve, which will increase the real interest rate. The increase in the real interest rate will lead to a second shift to the right on the labor supply curve since the relative cost of current leisure in terms of future leisure increases. All in all, the real wage decreases, the number of hours worked increases, the real interest rate increases and GDP increases as well (by less than the increase in G since the interest rate increases).

For a more complete justification see pages 372 to 377 of the textbook.

b)

$$y = r, x = (I, S)$$

As current taxes increase the lifetime wealth of the consumer decreases. However, due to the consumption smoothing behavior, consumption will decrease less than income. In this way, private savings will have to decrease. Since public savings $S^G = 0$, aggregate savings will be $S = S^P$. Given this, the aggregate savings curve will contract increasing the interest rate and decreasing aggregate Investment as well.
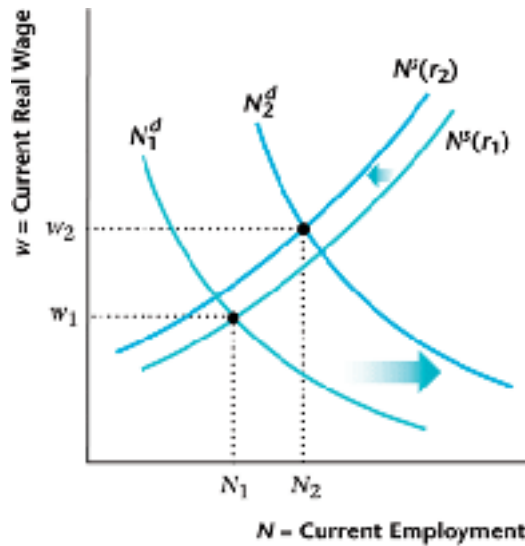
c) If the government decides to finance the increase in government expenditure through bonds, we know that, from the Ricardian Equivalence, future

taxes will increase. Since future taxes increase the future income of the consumer will decrease. In this way, to smooth consumption the consumer will have to increase savings in order to "transfer" income from the present to the future. All things considered, public savings will decrease, since the government issues debt while private savings increase since the consumer wants to smooth consumption across present and future.
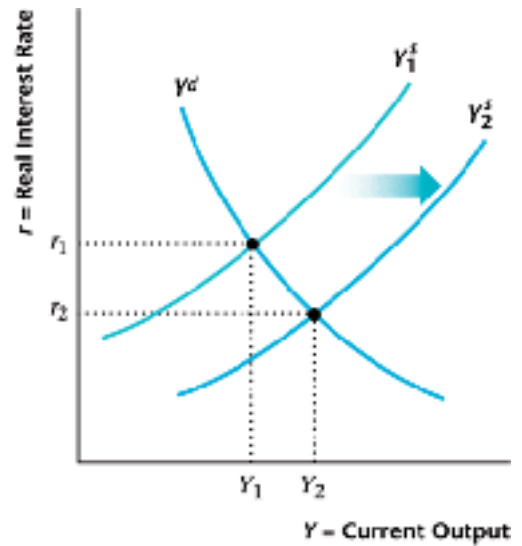
d) No, either way of financing the increase in public expenditure will lead to the same results in a). From the Ricardian equivalence we know that for a given present value of government expenditure the distribution of taxes to finance it between present and future is not relevant $(PV(G) = G + \frac{G'}{1+r} = T + \frac{T'}{1+r} = PV(T))$. The consumer knows that even if the government finances its current expenditures through debt, this will imply higher future taxes thus leaving him with the same lifetime wealth as if the government resorted to current taxes to finance the increaase in spending.
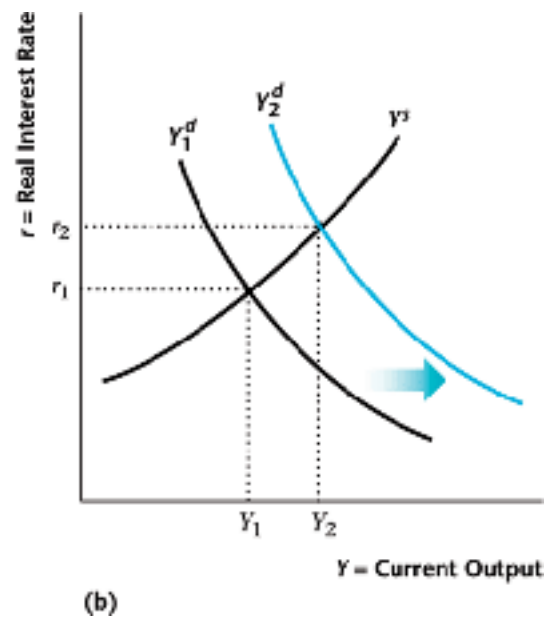
3.

a)

(a)   (b)

b)

3

N_1^d $N^s(r_1)$ $N^s(r_2)$

w_1

w_2

N_1 N_2

N = Current Employment

w = Current Real Wage

(a)

Y_1^d Y_2^d Y^s

r_2

r_1
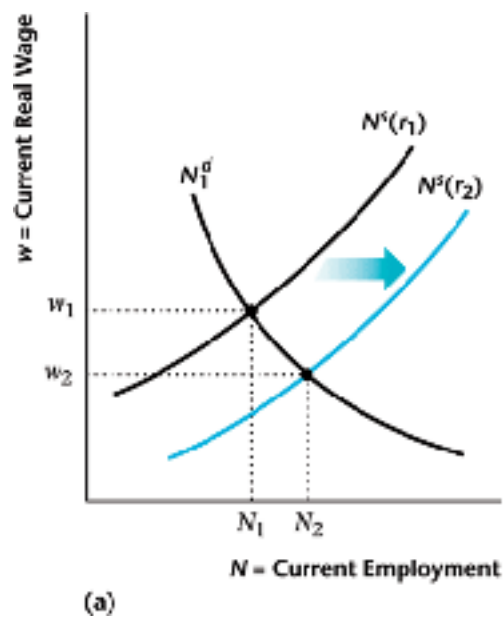
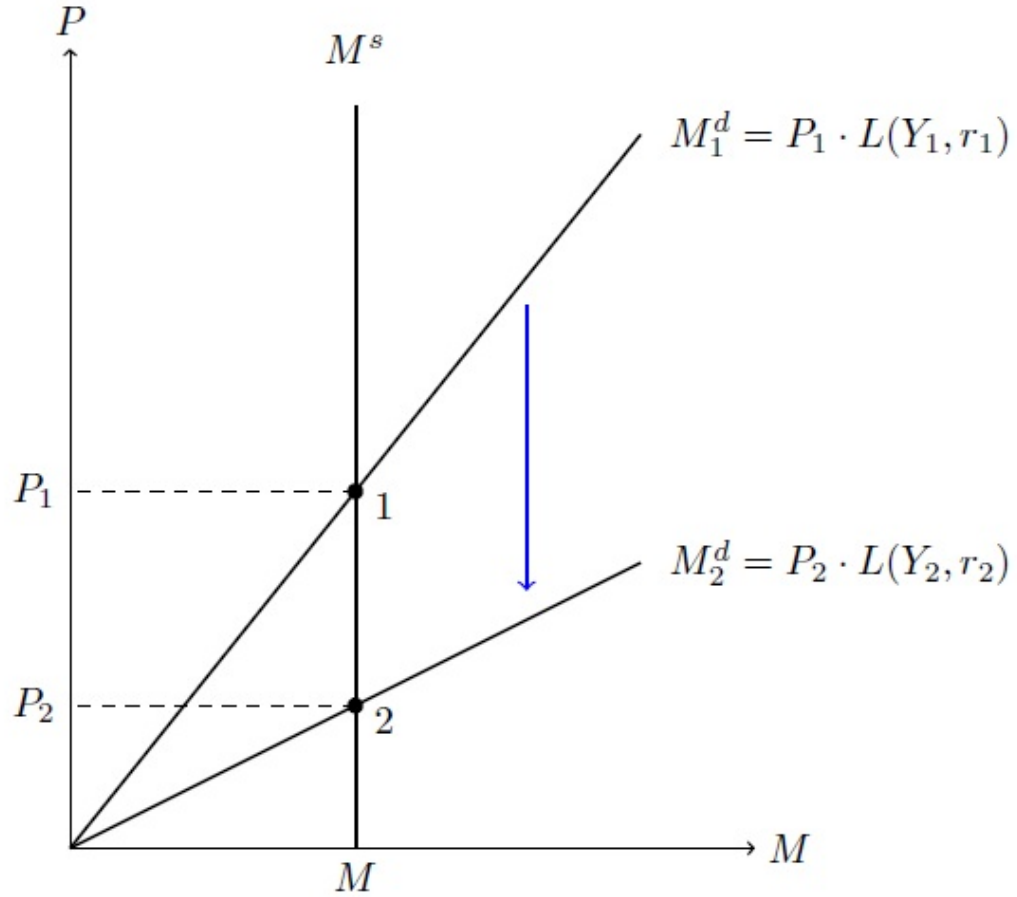Y_1 Y_2

Y = Current Output

r = Real Interest Rate

(b)

c)

4

As depicted in item a), an increase in current productivity will increase current output and decrease the real interest rate. Both the increase in output and the decrease in the real interest rate will increase money demand. This is so because for a higher income, consumers will wish to engage in a larger volume of transactions. In addition to this, a lower real interest rate implies a lower opportunity cost of holding money. In this way, for a constant money supply the price level will decrease.

4.

a) Since $N_t = 1$ in equilibrium we may express y_t in terms of (z_t, k_t) as follows:

$$y_t = z_t k_t^\alpha.$$

Regarding w_t and r_t we can also express them in terms of (z_t, k_t) through the solution to the representative firm's problem,

$$w_t = (1 - \alpha) z_t k_t^\alpha$$
$$r_t = \alpha z_t k_t^{\alpha-1}.$$

From the consumer's problem solution we already know that

$$k_{t+1} = \beta \alpha z_t k_t^\alpha.$$

Since $\delta = 1$, $i_t = k_{t+1}$, investment will have the same expression,

$$i_t = k_{t+1} = \beta \alpha z_t k_t^\alpha.$$

Finally, from the consumer's budget constraint we have that

$$c_t + k_{t+1} = w_t + r_t k_t$$
$$c_t = w_t + r_t k_t - k_{t+1}.$$

Replacing k_{t+1}, w_t and r_t we obtain

$$
\begin{aligned}
c_t &= (1 - \alpha) z_t k_t^\alpha + \alpha z_t k_t^{\alpha-1} k_t - \beta \alpha z_t k_t^\alpha \\
&= (1 - \alpha) z_t k_t^\alpha + \alpha z_t k_t^\alpha - \beta \alpha z_t k_t^\alpha \\
&= z_t k_t^\alpha - \beta \alpha z_t k_t^\alpha \\
&= (1 - \beta \alpha) z_t k_t^\alpha = (1 - \beta \alpha) y_t.
\end{aligned}
$$

b) The solution to the consumer's problem relates k_{t+1} to k_t as follows,

$$k_{t+1} = \beta \alpha z_t k_t^\alpha.$$

Replacing $z_t = 1$ we obtain

$$k_{t+1} = \beta \alpha k_t^\alpha,$$

the equation that describes the evolution of k_t for $z_t = 1$ for all t.

The value of k_t such that $k_t = k_{t+1}$ is the so called steady state value. Once capital reaches the steady state value, it remains constant at that point.

In this way for $k_t = k_{t-1} = k_{SS}$,

$$
\begin{aligned}
k_{SS} &= \beta \alpha k_{SS}^\alpha \\
k_{SS}^{1-\alpha} &= \beta \alpha \\
k_{SS} &= (\beta \alpha)^{1/(1-\alpha)}
\end{aligned}
$$

At the steady state capital is a function of β and α.

If k_0 is equal to k_{SS}, capital will be fixed. In this way, output, consumption, investment, the real wage and the real interest rate will be fixed as well.