Name: _____. Code: _____

Nova School of Business and Economics
Macroeconomics 1103, 2012-2013, 1st Semester
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TA: João Morgado

Problem Set 5
Due Date: Thursday, November 22, at the beginning of the class.
Once I start the class, I will not accept problem sets.

Turning in the problem sets is optional. For those that decide to turn in the problem sets, they have to be turned in on paper. The problem sets can be done in groups, but they have to be turned in individually.

Please, turn in your problem set with this cover page, with your name and code filled out above.

1. According to a news piece broadcast by TSF on November 2001, "Despite the crisis and the special tax over the Christmas subsidy this year, the Portuguese do not intend to cut significantly on the Christmas budget." Is this behavior in accordance to the observed for consumption over time? What does this imply about savings?

2. Consider an increase in current government expenditures.
a. What will be the effect of the increase in G on real wages, hours worked, real interest rates, and GDP? Show the effects with the $w \times N$ and $r \times Y$. Justify.

The budget constraint of the government in the current period is given by $G = T + B$, where T are taxes, charged lump sum, and B are government bonds. Government savings are given by $S^G = T - G$. Initially, $B = 0$. Aggregate savings in this economy is given by $S = S^P + S^G$, where S^P is private savings.

b. Consider that the increase in government expenditures is entirely financed by an increase in taxes. That is, $S^G = 0$ before and after the increase in government expenditures as $B = 0$ and $\Delta G = \Delta T$. What should happen with private savings after the increase in expenditures? Show the effects with a graph $r \times I, S$, with the interest rate in the vertical axis and investment and aggregate savings in the horizontal axis.
c. Suppose now that the government keeps taxes constant and finances the increase in government expenditures with an increase in bonds. What will be the effect on private savings? Explain.
d. Do the ways of financing the increase in government expenditures affect the results in a? Explain.

3. Obtain the effects of the following changes. Use pages 378-382 of the book.
a. An increase in current productivity, z.
b. An increase in future productivity, z'.
c. For the increase in current productivity in a, what are the predicted effects on the price level? Use the diagram $P \times M$ to explain.

4. (It is not necessary to turn in, but it will be useful for problem set 6.)
An economy has firms represented by a firm with production function $y_t = z_t N_t^{1-\alpha} k_t^{\alpha}$, where y_t is output, N_t is labor, k_t is capital, and z_t is productivity. Given wages w_t and interest rates r_t, the firm chooses capital and labor in order to solve the problem

$$\max_{N_t, k_t} z_t N_t^{1-\alpha} k_t^{\alpha} - w_t N_t - r_t k_t. \tag{1}$$

The firm will choose N_t and k_t to equalize the wage rate to the marginal productivity

of labor and the interest rate to the marginal productivity of capital. Therefore,

$$w_t = (1 - \alpha) z_t N_t^{-\alpha} k_t^{\alpha}, \tag{2a}$$
$$r_t = \alpha z_t N_t^{1-\alpha} k_t^{\alpha-1}. \tag{2b}$$

The consumers have preferences $\sum_{t=0}^{\infty} \beta^t \log c_t$. The consumers start a period with capital k_t and lend k_t to the firms. At the end of the period, the consumers receive labor income $w_t N_t$ and capital income $r_t k_t$, plus the lent capital with some depreciation. The budget constraint in each period is then

$$c_t + k_{t+1} = w_t N_t + r_t k_t + (1 - \delta) k_t, \tag{4}$$

where δ is depreciation of capital. To simplify, the consumers do not value leisure, and therefore they work all time available, $N_t = 1$. To simplify further, suppose that capital depreciates completely at the end of each period, $\delta = 1$. The budget constraint simplifies to

$$c_t + k_{t+1} = w_t + r_t k_t. \tag{5}$$

The problem of the consumer is then

$$\max_{c_t, k_t} \quad \sum_{t=0}^{\infty} \beta^t \ln c_t$$
$$\text{s.t.} \quad c_t + k_{t+1} = w_t + r_t k_t \quad t = 0, 1, 2, ..., \tag{6}$$

given a certain value for k_0, $k_0 > 0$.

The solution to this problem is such that

$$k_{t+1} = \beta \alpha z_t k_t^{\alpha}. \tag{7}$$

Investment at period t is given by $i_t = k_{t+1} - (1 - \delta) k_t$. As $\delta = 1$, we have $i_t = k_{t+1}$.

a. Use the solution above for the problem of the consumer and for the problem of the firm to write y_t, w_t, r_t, k_{t+1}, i_t, and c_t as a function of z_t and k_t. Remember that $N_t = 1$ in equilibrium.

b. Given k_t, capital k_{t+1} will be, in general, different than k_t. Suppose that z_t is constant over time. Describe how k_t should evolve over time for $z_t = 1$ for all t and given a small value for the initial capital k_0, $k_0 > 0$. Is there a value of k such that $k_{t+1} = k_t$? How does the economy behave if k_0 is equal to this value?