Nova School of Business and Economics
Macroeconomics 1103, 2012-2013, 1st Semester
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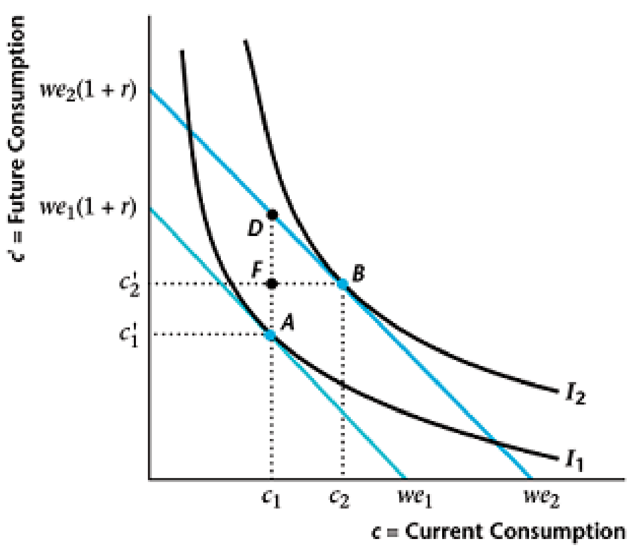
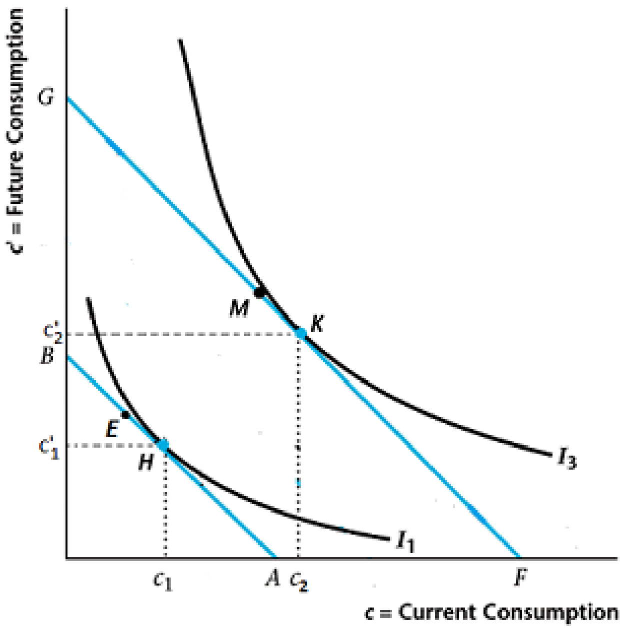
Problem Set 4
TA Solution

1.

a) The increase in y_2 is represented by the following graph:

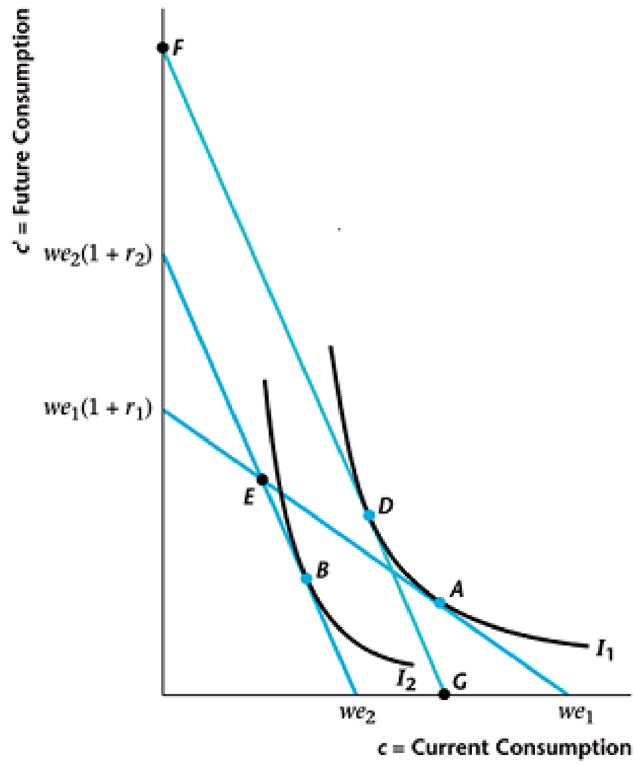


Before the increase in y_2 the consumer chooses the consumption bundle A, after future income increases the optimal choice moves to point B. As the graph clearly shows both current and future consumption increase from (c_1,c_2) to ($c_1^{'}$,$c_2^{'}$). The increase in future consumption, corresponding to the distance AF is smaller than the increase in future income corresponding to the distance AD. This is the case because the consumer wants to smooth consumption across the two periods, thus increasing consumption in the future less than the increase in income while decreasing savings in the current period.



The graph above shows the case in which both current and future income increase with the endowment point moving from point E to point M. In this case, current and future consumption will increase as the initial and final choice (H and K) depict. The difference is that the variation will be much more significant now. Furthermore, savings will have a lower a variation or not vary at all when compared to the last case since the consumer will finance the increase in current consumption (or at least a large part of it) with the additional current income.

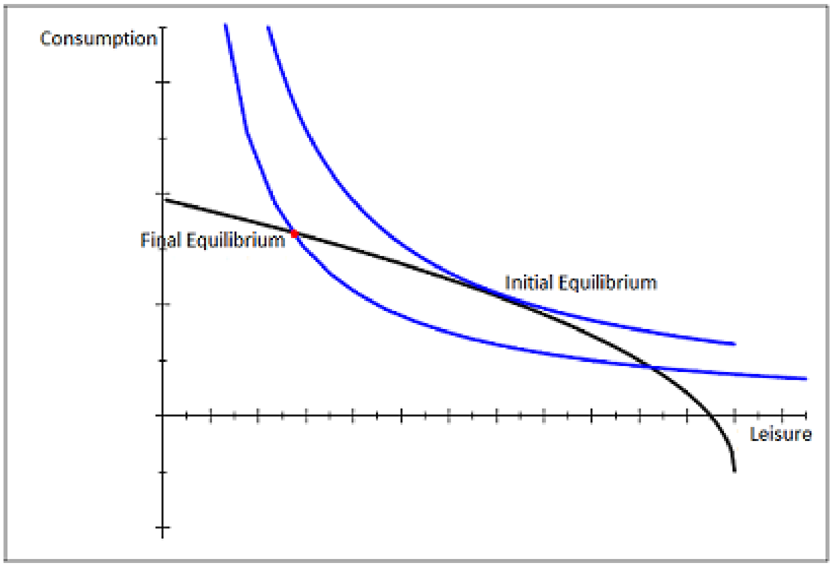
b) The following graph depicts the increase in the real interest rate for a borrower:



The inicial choice of the consumer corresponds to point A while the final choice (after the increase in r) corresponds to point B. The effect is divided between substitution and income effect, the first corresponding to the movement AD and the second to the movement DB. The substitution effect (AD) is negative for current consumption and positive for future consumption. This effect does not change whether we are dealing with a lender or a borrower, an increase in the interest rate always increases the relative cost of current consumption in terms of future consumption. The income effect though, will be different between lenders and borrowers. When the consumer is a borrower, the income effect is negative for both current and future consumption. Taking both effects together, we know that current consumption will fall while future consumption may rise or fall depending on the relative size of the substitution and income effects. Since current income did not change, savings must rise as well.

2.

The following graph shows the equilibrium of this economy before and after the introduction of the proportional subsidy to the workers/consumers.



With the subsidy the budget constraint faced by consumers becomes $c = 1.1w(h-l) + \pi - T$. In this way, the introduction of a subsidy proportional to labor income increases the marginal benefit from working an extra hour. Given this, the consumers will choose less leisure and increase labor until $MRS_{l,c}$ is equal to the subsidized real wage. Since the labor supplied by workers/consumers increased, the marginal productivity of labor will decrease. This in turn will lead to an ajustment in the real wage (without the subsidy) since firms always set $MP_N = w$ in equilibrium. Intuitively, this is the same as saying that the firms will only accept to hire more labor for a lower wage. To sum up, labor and the wage received by the workers will increase (due to the subsidy received from the government) while the wage paid by firms will decrease simultaneously (since MP_N will decrease).

a) The subsidy will increase production since it will increase labor for a given level of capital (K) and total factor productivity (z). ($Y = zF\left(K, \underset{+}{N}\right)$)

b) Since the final equilibrium lies in a lower indifference curve than the initial equilibrium it is clear that the measure will decrease welfare.

3.

a) True. A temporary increase in income will have a stronger impact on savings since consumers want to smooth consumption. When faced with a temporary increase in their income consumers will consume a small part of it while saving the rest to increase consumption in the future as well. When, in alternative, the consumers are faced with a permanent increase in their income they will increase current consumption by a larger amount given that their future income will also increase. In this way, savings will increase less or even decrease.

b) Yes, savings can fluctuate over time. Several different factors may explain this fluctuations namely any variation in the interest rate r, the discount factor β, current or future income (y_1, y_2). Any shock that affect this variables will also have an impact on the savings rate, leading to fluctuations in savings over time.

c) False. As we know consumers like to smooth consumption. In this way, if a consumer knows in advance that his income will decrease over time, he will increase present savings (thus decreasing current consumption) in order to keep consumption roughly constant between present and future.

4.

a) As explained in the previous problem set, from the budget constraints of both periods we can derive the intertemporal budget constraint:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}.$$

Maximizing the utility function subjected to this restriction we obtain the optimal levels of consumption in each period:

$$\begin{aligned} c_1^* &= \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r}\right), \\ c_2^* &= \frac{\beta}{1+\beta}\left[(1+r)\,y_1 + y_2\right]. \end{aligned}$$

Given the restriction for $t = 1$, $s = y_1 - c_1$, we obtain the optimal level of savings replacing c_1 by its optimal level c_1^* as follows:

$$\begin{aligned} s^* &= y_1 - c_1^* \\ &= y_1 - \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r}\right) \\ &= \frac{\beta y_1}{1+\beta} - \frac{y_2}{(1+r)(1+\beta)}. \end{aligned}$$

b) In the absence of investment and public debt, aggregate savings will be equal to zero in equilibrium, $S = 0$. The equilibrium interest rate is the one that garantees that this relation is verified. Recall that agreggate savings are equal to the sum of the individual optimal savings of each of the N consumers in this economy

$$S = \sum_{i=1}^{N} s_i^*.$$

Given that all consumers share the same preferences and endowment (y_1, y_2) we know that $s_i^* = s^*, \forall i$, this is the optimal savings will be the same for all

the N consumers. In this way, in equilibrium we will have

$$\begin{aligned}
\sum_{i=1}^{N} s_i^* &= N s^* = 0 \\
s^* &= 0 \\
\frac{\beta y_1}{1+\beta} - \frac{y_2}{(1+r)(1+\beta)} &= 0 \\
1 + r^* &= \frac{y_2}{\beta y_1}.
\end{aligned}$$

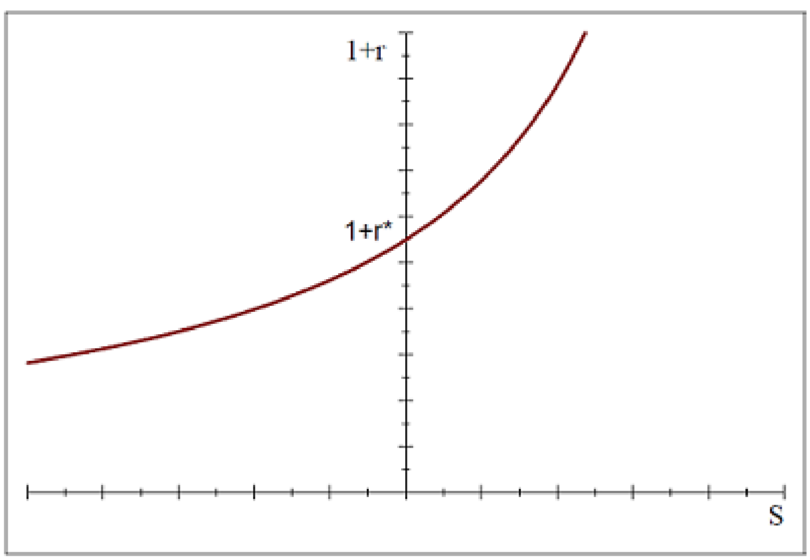
Concerning the savings curve we can obtain it by rewriting optimal savings, s^*, in order to $1+r$:

$$1 + r = \frac{y_2}{\beta y_1 - (1+\beta) s^*}$$

Regarding the slope of the curve, we can find whether it is positive or not through the sign of the following derivative:

$$\frac{\partial (1+r)}{\partial s^*} = \frac{y_2 (1+\beta)}{\left[\beta y_1 - (1+\beta) s^*\right]^2} > 0.$$

This proves there is a positive relation between the interest rate and savings. The curve is represented in the graph bellow:

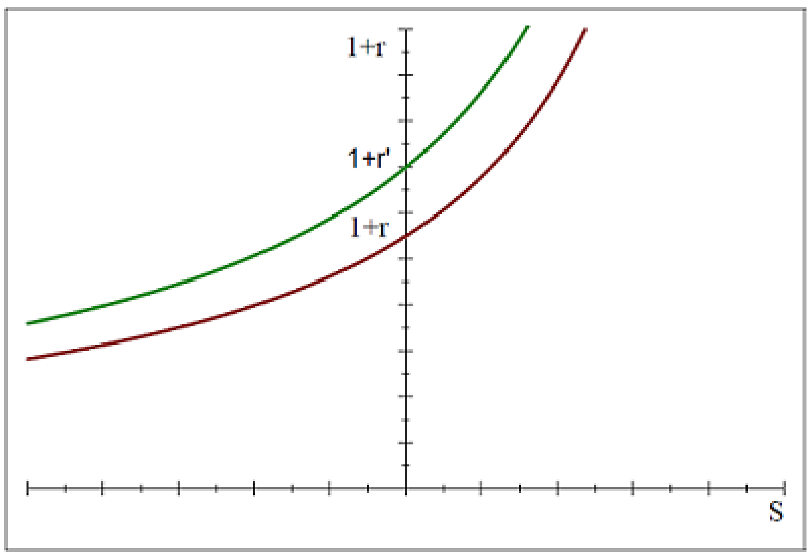


c) An increase in y_2 implies an increase in the intertemporal wealth of the consumer. Since both c_1 and c_2 are normal goods consumption will increase

in both periods. Given that current consumption is increasing while current income remained fixed, savings will decrease for the same interest rate. In other words, there will be less consumers willing to lend while more consumers will want to borrow. For the aggregate savings to remain equal to zero the interest rate will have to increase in order to correct for this imbalance. In practice, the increase in the interest rate will encourage consumers to lend more and borrow less. Note that:

$$\begin{aligned} \frac{\partial s^*}{\partial y_2} &= -\frac{1}{(1+r)(1+\beta)} < 0, \\ \frac{\partial (1+r^*)}{\partial y_2} &= \frac{1}{\beta y_1} > 0. \end{aligned}$$

This shock corresponds to the following graphical representation:



d) We are told that the interest rate is $4\% \Rightarrow r = 0.04$ and that the output growth rate is $2\% \Rightarrow y_2/y_1 = 1.02$ (since everyone has the same endowment, the ratio between current and future individual income must be equal to output growth). Assuming the credit market is in equilibrium, we know that

$$1 + r = \frac{y_2}{\beta y_1}.$$

Replacing by the figures given to us we obtain

$$\begin{aligned} 1.04 &= \frac{1.02}{\beta} \\ \beta &= \frac{1.02}{1.04} = 0.98. \end{aligned}$$

Thus, the value of β compatible with this observations is 0.98.

5.

a) In this model the government is restricted by the following budget constraint:

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}.$$

The present value of government expenditure is always equal to the present value of taxes (the government always pays back the debt issued). If the government decides to set $T_1 = 0$ while keeping the expenditure fixed in both periods, than for the budget constraint to hold taxes will have to increase in the second period. ($|\Delta T_1| = |\Delta \frac{T_2}{1+r}|$)

According to the Ricardian equivalence the timing of taxes for a given level of government expenditures has no effect on the real interest rate and on the optimal choice of the consumer. This is the case because the consumer realises that a tax cut in the present will be offset by a tax raise in the future, thus leaving his intertemporal wealth unchanged. Since neither the wealth nor the preferences of the consumer change, his optimal choice will not change as well. Given that the optimal choice will not change, consumption at time 1 will remain the same.

b) If taxes decrease to 0 in the first period the net current income of the consumer will increase. Since consumption does not change, savings will increase by the exact amount of the tax cut. This increase in savings will allow the consumer to keep the same level of consumption in second period when taxes increase again.

6.

a) The consumer optimization problem is given by

$$\begin{aligned}
& \max_{c_1,l_1,c_2,l_2} \{\log c_1 + \log l_1 + \beta(\log c_2 + \log l_2)\} \\
& s.t \\
c_1 + \frac{c_2}{1+r} &= z_1(h-l_1) + \frac{z_2(h-l_2)}{1+r} - T_1 - \frac{T_2}{1+r}.
\end{aligned}$$

The Lagrange function will be

$$\begin{aligned}
L &= \log c_1 + \log l_1 + \beta(\log c_2 + \log l_2) \\
&\quad +\lambda\left[z_1(h-l_1) + \frac{z_2(h-l_2)}{1+r} - T_1 - \frac{T_2}{1+r} - c_1 - \frac{c_2}{1+r}\right].
\end{aligned}$$

First order conditions:

$$\begin{aligned}
\frac{\partial L}{\partial c_1} &= 0 \Leftrightarrow \frac{1}{c_1} = \lambda \\
\frac{\partial L}{\partial l_1} &= 0 \Leftrightarrow \frac{1}{l_1} = \lambda z_1 \\
\frac{\partial L}{\partial c_2} &= 0 \Leftrightarrow \frac{\beta}{c_2} = \frac{\lambda}{1+r} \\
\frac{\partial L}{\partial l_2} &= 0 \Leftrightarrow \frac{\beta}{l_2} = \frac{\lambda z_2}{1+r} \\
\frac{\partial L}{\partial \lambda} &= 0 \Leftrightarrow c_1 + \frac{c_2}{1+r} = z_1 (h - l_1) + \frac{z_2 (h - l_2)}{1+r} - T_1 - \frac{T_2}{1+r}.
\end{aligned}$$

b) From the first order conditions, we have

$$\begin{aligned}
\frac{c_2}{c_1} &= \frac{\frac{\beta(1+r)}{\lambda}}{\frac{1}{\lambda}} = \beta (1+r), \\
\frac{l_2}{l_1} &= \frac{\frac{\beta(1+r)}{\lambda z_2}}{\frac{1}{\lambda z_1}} = \frac{z_1}{z_2} \beta (1+r).
\end{aligned}$$

When the interest rate increases, $(c_2/c_1) \uparrow$ and $(l_2/l_1) \uparrow$. That is, both consumption and leisure grow from the first period to the second. To begin with, a higher interest rate makes future consumption cheaper in relative terms, thus encouraging the consumer to reduce current consumption and increase future consumption. In addition to this, a higher interest rate increases the return on savings. In this way, the consumer will be encouraged to accumulate more income in the present in order to increase savings. To accumulate more income the consumer will work more in the present, thus substituting current leisure for future leisure.

c)

Using the first order conditions, rewrite c_1, l_1, c_2 and l_2 in terms of λ. We obtain

$$\begin{aligned}
c_1 &= \frac{1}{\lambda}, \\
l_1 &= \frac{1}{\lambda z_1}, \\
c_2 &= \frac{\beta (1+r)}{\lambda}, \\
l_2 &= \frac{\beta (1+r)}{\lambda z_2}.
\end{aligned}$$

We find the value of λ with respect to the exogenous variables of the problem replacing c_1, l_1, c_2 and l_2 on the budget constraint, the last first order condition:

$$c_1 + \frac{c_2}{1+r} = z_1 (h - l_1) + \frac{z_2 (h - l_2)}{1+r} - T_1 - \frac{T_2}{1+r}.$$

$$\Rightarrow \quad \frac{1}{\lambda}+\frac{\frac{\beta(1+r)}{\lambda}}{1+r}=z_1h-z_1\frac{1}{\lambda z_1}+\frac{z_2h-z_2\frac{\beta(1+r)}{\lambda z_2}}{1+r}-T_1-\frac{T_2}{1+r}$$
$$\Rightarrow \quad \frac{1}{\lambda}+\frac{\beta}{\lambda}=z_1h-\frac{1}{\lambda}+\frac{z_2h}{1+r}-\frac{\beta}{\lambda}-T_1-\frac{T_2}{1+r}$$

$$\Rightarrow \lambda=\frac{2\left(1+\beta\right)}{z_1h-T_1+\frac{z_2h-T_2}{1+r}}.$$

Replacing λ in the expressions of c_1, l_1, c_2 and l_2 we obtain their optimal values with respect to $(r,z_1,z_2,T_1,T_2,\beta)$,

$$\begin{aligned} c_1^* &= \frac{1}{2\left(1+\beta\right)}\left(z_1h-T_1+\frac{z_2h-T_2}{1+r}\right), \\ l_1^* &= \frac{1}{2z_1\left(1+\beta\right)}\left(z_1h-T_1+\frac{z_2h-T_2}{1+r}\right), \end{aligned}$$

$$\begin{aligned} c_2^* &= \frac{\beta\left(1+r\right)}{2\left(1+\beta\right)}\left(z_1h-T_1+\frac{z_2h-T_2}{1+r}\right), \\ l_2^* &= \frac{\beta\left(1+r\right)}{2z_2\left(1+\beta\right)}\left(z_1h-T_1+\frac{z_2h-T_2}{1+r}\right). \end{aligned}$$

From l_1^* and l_2^* we obtain the labor supply for each period:

$$\begin{aligned} N_1^s &= h-l_1^*, \\ N_2^s &= h-l_2^*. \end{aligned}$$

Taking the first derivative of c_1^* with respect to the interest rate r we obtain:

$$\frac{\partial c_1^*}{\partial\left(1+r\right)}=-\frac{z_2h-T_2}{2\left(1+\beta\right)\left(1+r\right)}<0.$$

Thus, when the interest rate increases, current consumption decreases (the substitution effect is stronger than the income effect in this case).

Since the labor demand curve is perfectly elastic, N_1 will be determined by labor supply. Taking the first derivative of N_1^s with respect to the interest rate r we obtain:

$$\begin{aligned} \frac{\partial N_1}{\partial\left(1+r\right)} &= -\frac{\partial l_1^*}{\partial\left(1+r\right)} \\ &= \frac{z_2h-T_2}{2z_1\left(1+\beta\right)\left(1+r\right)}>0 \end{aligned}$$

Thus, labor increases when the interest rate increases. This happens because the consumer wants to work more to increase savings when the interest rate increases.