Nova School of Business and Economics
Macroeconomics 1103, 2012-2013, 1st Semester
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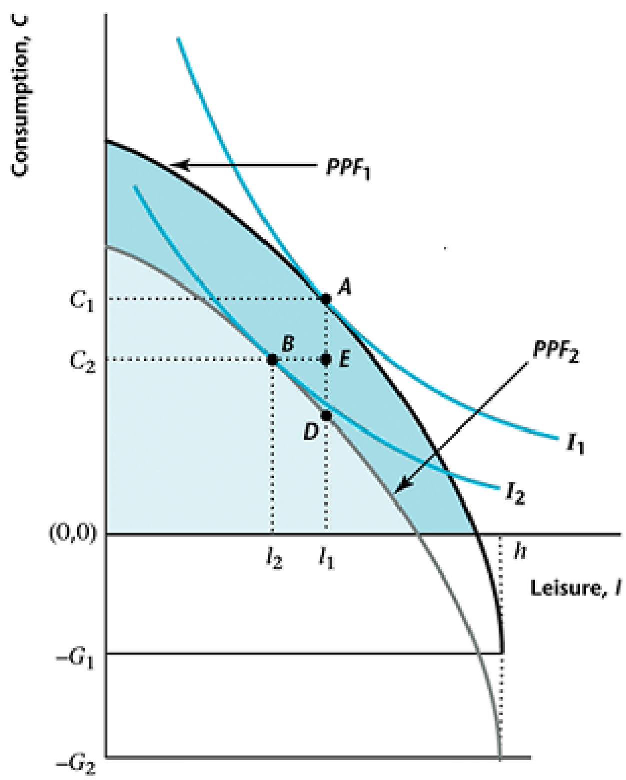
Problem Set 3
TA Solution

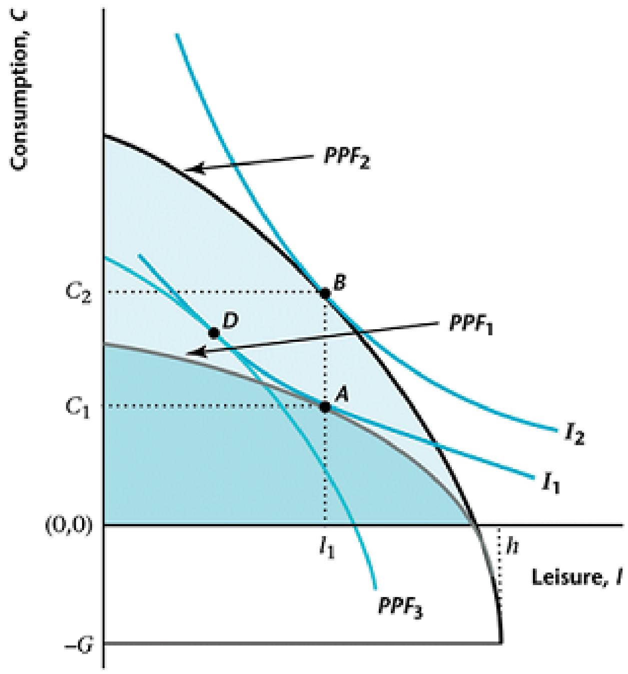
1.

a) Increase in government expenditures.



Given the increase in government expenditures, both leisure and consumption will decrease as depicted in the graph above (compare points A and B). Since leisure decreases, labor will have to increase. Furthermore, given that both z and K remain fixed while labor increases, output will increase as well. Regarding welfare, it will decrease since the final equilibrium lies in a lower indifference curve. Finally, in what concerns the real wage we know that in equilibrium $MP_N = w$ and that $\frac{\partial MP_N}{\partial N} < 0$ (MP_N is decreasing in N). In this way, since labor increased, the marginal productivity of labor will decrease and so will the equilibrium real wage.

b) Increase in total factor productivity, z.



Given an increase in z, the equilibrium will move from point A to point B as the graph shows. From point A to point B consumption will definitely increase. Regarding the effect on leisure, we will divide it into substitution and income effect. The two effects have opposing impacts on leisure, while the substitution effect (A-D) shows a decrease in leisure, the income effect (D-B) has a positive impact on the variable. In the graph presented, the income effect perfectly offsets the substitution effect leaving leisure unchanged, however we can not take any conclusion on the net effect of an increase of z on leisure. Given that a change in z has an ambiguous effect on leisure, the impact on labor, N, will be ambiguous as well. Concerning output, given the income-expenditure identity, $Y = C+G$, we know that it will increase since government expenditure remains fixed as consumption increases. As the graph shows, the impact on welfare is positive since the consumer ends up in a higher indifference curve. Finally, the equilibrium wage will increase. To explain why we will resort to the substitution and income effects once again. From A to D the consumer will move along the initial indifference curve to a point, D, with more consumption and less leisure. In equibrium the condition $MRS_{l,c} = w$ is always verified, this is the marginal rate of substitution or the slope of the indifference curve is equal to the equilibrium real wage. Since $MRS_{l,c}$ increases as the quantity of leisure decreases we know that w will increase from A to D. From point D to point B (the income effect) the equilibruim moves from PPF_3 to PPF_2 (the intermediate and final production possibilities frontier curves which are parallel) to a point with more leisure and consumption. In equilibrium we know that $MP_N = w$, the slope of the PPF is equal to equilibrium wage. Since MP_N increases with

leisure (more leisure implies less labor thus higher marginal productivity of this input), from D to B w will increase as well. Taking both effects together, it is clear that an increase in z has a positive impact on w.

c) Empirical analysis of business cycles tells us that consumption, labor and the real wage are procyclical variables, in other words the deviations from trend of these variables are positively correlated with the deviations from trend in GDP. To conclude whether the fluctuations in government spending, G, or the fluctuations in total factor productivity, z, are more likely to explain business cycles we must compare the predictions of our model to the patterns observed empirically. From item a) we know that when G increases, output and labor increase as well. However, consumption and the real wage will decrease which is not consistent with the data. In this way, the fluctuations in G are not a good explanation of business cycles. Regarding the fluctuations of z we know from item b) that when z increases, output, consumption and the real wage will increase too. Even though, the effect on labor is ambiguous, it may increase or decrease depending on the relative size of the substitution and income effects. Despite this fact, the fluctuations of total factor productivity are likely to be one of the main explanations (if not the most important) of business cycles.

2.

a) Since the consumer owns the firm his maximization problem will be restricted by the production possibilities frontier instead of the budget constraint. Given that there is no government in this model and that $N = h - l$ the PPF corresponds to $C = z(h - l)$. In this way, the optimization problem is given by:

$$\begin{aligned} & \max_{l,c} \quad \log c + \log l && (1) \\ & s.t. && \\ c \quad = \quad & z\left(h-l\right). && (2) \end{aligned}$$

The lagrange function corresponds to:

$$L = \log c + \log l + \lambda\left[z\left(h-l\right) - c\right]. \tag{3}$$

Taking the first order conditions we obtain:

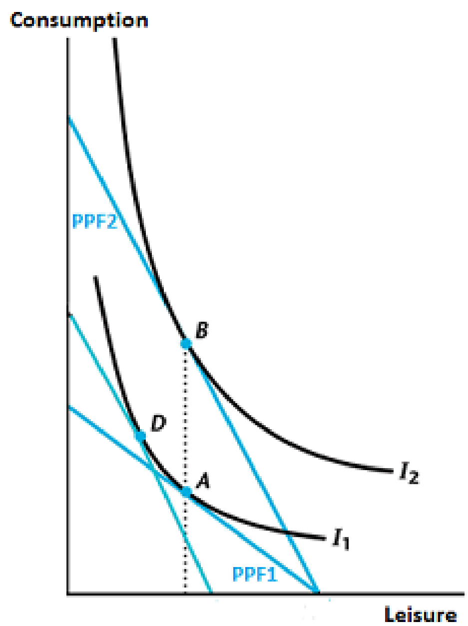
$$\begin{cases} \dfrac{\partial L}{\partial l} = \dfrac{1}{l} - \lambda z = 0 \\ \dfrac{\partial L}{\partial c} = \dfrac{1}{c} - \lambda = 0 \\ \dfrac{\partial L}{\partial \lambda} = z\left(h-l\right) - c = 0 \end{cases} \Leftrightarrow \begin{cases} \dfrac{1}{l} = \lambda z \\ \dfrac{1}{c} = \lambda \\ z\left(h-l\right) = c \end{cases}. \tag{4}$$

Dividing the first two conditions we get:

$$\begin{cases} \dfrac{c}{l} = z \\ \\ z\,(h-l) = c \end{cases} \Leftrightarrow (...) \Leftrightarrow \begin{cases} c^* = \dfrac{zh}{2} \\ \\ l^* = \dfrac{h}{2} \end{cases}. \tag{5}$$

Taking the optimal choice of leisure, l^*, we can derive the optimal choice of labor, $N^* = h - l^* = h - \dfrac{h}{2} = \dfrac{h}{2}$.

b)



The graph above shows the impact of an increase in z in this model (as the graph depicts in this model the PPF is linear). The shock (A-B) is divided into substitution and income effect. The substitution effect corresponds to the movement from A to D and the income effect corresponds to the movement from D to B. As the graph shows leisure remains unchanged between the initial and final equilibria while consumption increases, this implies that the income effect perfectly offsets the substitution effect. This can be demonstrated taking the first derivatives of the optimal choices of consumption and leisure in order to z:

$$\frac{\partial c^*}{\partial z} = \frac{h}{2} > 0,$$

$$\frac{\partial l^*}{\partial z} = 0.$$

c) The optimal consumption in discrete time (period t) is given by the following expression:

$$c_t^* = \frac{z_t h}{2}. \tag{6}$$

Given that total factor productivity grows according to the following pattern,

$$z_{t+1} = (1+g)z_t, \tag{7}$$

consumption in period $t+1$ will correspond to

$$c_{t+1}^* = \frac{(1+g)z_t h}{2}. \tag{8}$$

Since the growth rate might be estimated as

$$g_{c^*} = \log\left(c_{t+1}^*\right) - \log\left(c_t^*\right). \tag{9}$$

Replacing the expressions we get

$$\begin{aligned}
g_{c^*} &= \log\left[\frac{(1+g)z_t h}{2}\right] - \log\left[\frac{z_t h}{2}\right] \\
&= \log[(1+g)z] + \log(h) - \log(2) - \log(z) - \log(h) + \log(2) \\
&= \log(1+g) + \log(z) - \log z = \log(1+g) \simeq g.
\end{aligned}$$

The growth rate of consumption is equal to the growth rate of total factor productivity.

Concerning labor, its growth rate will be 0% since the optimal choice, $\frac{h}{2}$, does not depend on total factor productivity, which is the only parameter that grows over time.

d) Since the consumer no longer owns the firm, the representative consumer and representative firm will make their optimal decisions (on labor, leisure and consumption) separately.

The optimization problem faced by the firm corresponds to:

$$\max_N \ \pi = zN - wN. \tag{10}$$

Taking the first order condition we obtain:

$$\frac{\partial \pi}{\partial N} = z - w = 0 \Leftrightarrow z = w.$$

From the first order condition we know that the labor demand curve will be an horizontal line for a real wage equal to z. Given this, the equilibrium wage will always be equal to z.

Since that z, the total factor productivity, grows at a rate g, the equilibrium wage will grow at the same rate.

3.

With taxes proportional to income the budget constraint of the consumer becomes

$$c = w\left(1-\tau\right)N + \pi.$$

In terms of labor and leisure the budget constraint corresponds to

$$c = w\left(1-\tau\right)\left(h-l\right) + \pi.$$

Regarding the production function, it takes the same form as in the previous exercise

$$Y = zN.$$

In this way, in equilibrium two different conditions will hold. The firm will set

$$MP_N = w \Leftrightarrow z = w,$$

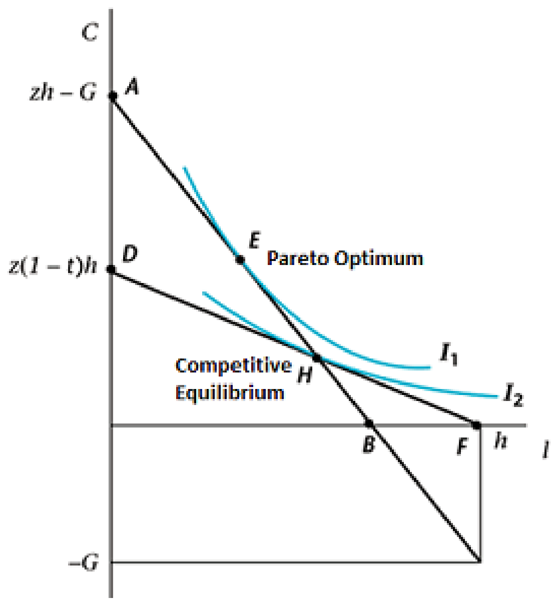
while the consumer will set

$$MRS_{l,c} = w\left(1-\tau\right).$$

Note that the consumer is no longer setting the marginal rate of substitution equal to the real wage, w, because for $\tau > 0$, $w\left(1-\tau\right) < w$. This implies that the relative price of leisure in terms of consumption is now lower since the consumer receives less money for each additional hour he works. Despite this fact, the firm still pays a wage w for each hour worked, as the firm´s optimal condition demonstrates. In this way, in equilibrium the representative consumer and representative firm will not have the same optimizing conditions ($MRS_{l,c}$ will be different from MP_N) what will cause a distortion in the economy. As a consequence, the equilibrium in this economy will be different from the Pareto optimum.

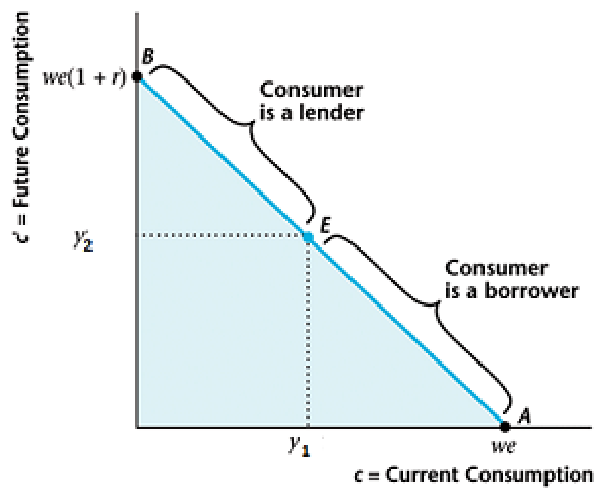
What are the pratical implications of this distortion? Since the relative price of leisure is now lower, the consumer will choose more leisure and less consumption than what he would if he was optimizing for the real wage without taxes, w. Furthermore, since leisure will be higher, labor supply, N^s, will necessarily be lower. As a consequence output, Y, will decrease as well.

The following graph depicts this problem:



4.

a)



When the optimal choice of the consumer is point E, corresponding to his endowment point (y_1, y_2), the consumer will be neither a lender nor a borrower. When he chooses a point to the right of E, $c_1 > y_1 \Rightarrow s < 0$, thus he will be a borrower. If the consumer chooses a point to the left of E, $c_1 < y_1 \Rightarrow s > 0$, thus he will be a lender.

b) The optimal levels of consumption and savings are obtained through the resolution of the consumer´s optimization problem. To solve the optimization problem we need to achieve the expression that gives us the intertemporal

budget constraint (IBC). From the budget constraints of the first and second period we may derive the IBC as:

$$c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r}.$$

In this way the optimization problem will be given by:

$$\begin{aligned} & \max_{c_1,c_2} \{\log c_1 + \beta \log c_2\} \\ & s.a \\ c_1 + \frac{c_2}{1+r} &= y_1 + \frac{y_2}{1+r} \end{aligned}$$

Taking the first order conditions we get

$$\begin{cases} \dfrac{\partial L}{\partial c_1} = \dfrac{1}{c_1} - \lambda = 0 \\ \\ \dfrac{\partial L}{\partial c_2} = \dfrac{\beta}{c_2} - \dfrac{\lambda}{1+r} = 0 \\ \\ \dfrac{\partial L}{\partial \lambda} = y_1 + \frac{y_2}{1+r} - c_1 - \frac{c_2}{1+r} = 0 \end{cases} \Leftrightarrow \begin{cases} \dfrac{1}{c_1} = \lambda \\ \\ \dfrac{\beta}{c_2} = \dfrac{\lambda}{1+r} \\ \\ c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \end{cases}. \tag{11}$$

Dividing the first two conditions we obtain,

$$\begin{cases} \frac{c_2}{\beta c_1} = 1 + r \\ \\ c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \end{cases} \Leftrightarrow \begin{cases} c_2 = (1+r)\,\beta c_1 \\ \\ c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} \end{cases}. \tag{12}$$

Solving for c_1 and c_2,

$$\begin{cases} c_1^* = \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r}\right) \\ \\ c_2^* = \frac{\beta}{1+\beta}\left[(1+r)\,y_1 + y_2\right] \end{cases} \tag{13}$$

To obtain the optimal level of savings we depart from the budget constraint of the first period,

$$c_1 + s = y_1 \Leftrightarrow s = y_1 - c_1.$$

Replacing by c_1^* in the expression we get,

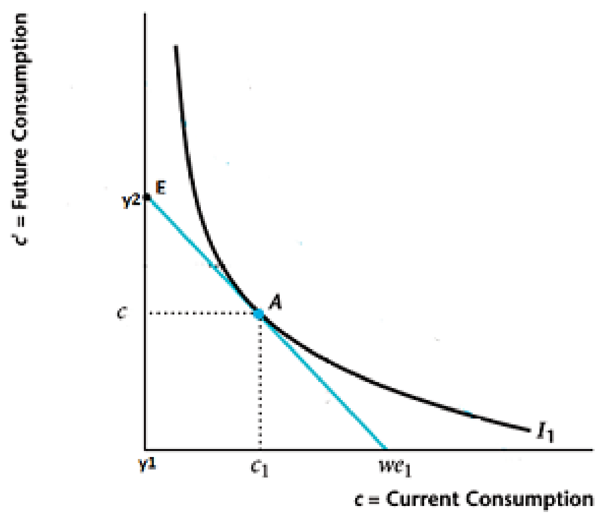
$$\begin{aligned} s^* &= y_1 - \frac{1}{1+\beta}\left(y_1 + \frac{y_2}{1+r}\right) \Leftrightarrow \\ &\Leftrightarrow \; s^* = \frac{\beta}{1+\beta} y_1 - \frac{y_2}{(1+\beta)(1+r)} = \frac{\beta y_1 - \frac{y_2}{1+r}}{1+\beta} \end{aligned}$$

c) Replacing $y_1 = 0$ in the expressions of c_1^*, c_2^* and s^* we immediately understand if the consumer is a borrower or not:

$$\begin{aligned} s^* &= \frac{-\frac{y_2}{1+r}}{1+\beta} \\ c_1^* &= \frac{1}{1+\beta}\left(\frac{y_2}{1+r}\right) \\ c_2^* &= \frac{\beta}{1+\beta} y_2. \end{aligned}$$

The consumer is clearly a borrower since savings are negative and consumption in period 1 is positive. This makes sence because the consumer wants to smooth consumption, redistributing his income from period 2 over the two periods.

The following graph shows this situation:



d) To determine the impact of an increase in y_1 on s^* and c_2^* we just need to analyse the sign of the first derivative of these two variables as follows:

$$\frac{\partial s^*}{\partial y_1} = \frac{\beta}{(1+\beta)} > 0$$

$$\frac{\partial c_2^*}{\partial y_1} = \frac{\beta}{1+\beta}(1+r) > 0$$

Since the derivatives are positive for both s^* and c_2^* we can conclude that whenever y_1 increases savings and consumption will increase as well. Since the consumer wants to smooth consumption he will consume part of the additional

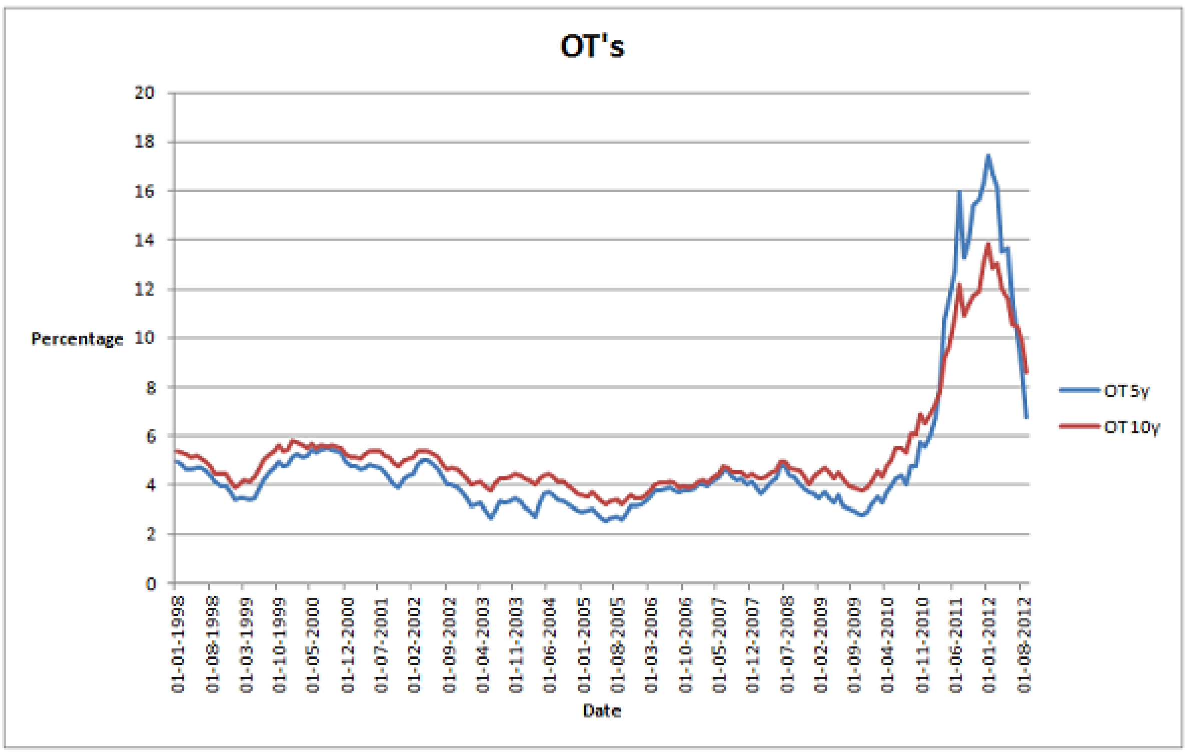
income he receives and save the rest to increase consumption in the future as well.

The level of y_1 for which the consumer switches from a borrower to a lender can be estimated by setting $s^* = 0$ and solving the equation in order to y_1.

$$s^* = \frac{\beta y_1 - \frac{y_2}{1+r}}{1+\beta} = 0 \Leftrightarrow y_1 = \frac{y_2}{\beta(1+r)}.$$

Taking the last derivation for a level of y_1 higher than $\frac{y_2}{\beta(1+r)}$ the consumer will switch from borrower to lender.

5.



The graph above shows the evolution of the interest rate on Portuguese Government bonds (obrigações do tesouro) with maturities of 5 and 10 years. The data was extracted from BPstat, the observations are monthly and report to a period between January 1998 and September 2012. As the graph clearly shows, the interest rates on the 5 and 10 years bonds move closely together across the whole period. Between 1998 and 2010 the interest rates for both bonds where substantially low moving roughly between 3 and 6%. In addition, across this period the interest rate on the bonds of 10 years was always higher

than the interest rate on the bonds of 5 years. This difference is common since investors usually perceive bonds with longer maturities as riskier. From 2010 to the present though, the behavior of the interest rates on the Portuguese Government bonds completly changed. To begin with, the interest rate on both the 5 years and 10 years bonds raised very significantly between 2010 and the beggining of the current year, in fact by January 2012 the interest rate on the bonds of 5 years reached 17,42%. This was a consequence of the current financial crisis which increased the risk of the Portuguese bonds. Moreover, between March of 2011 and June of 2012, the interest rates on the bonds of 5 years were higher than the interest rates on the bonds of 10 years. A possible explanation is that investors might expect the Portuguese economy to recover in the long-term while regarding its short-term situation as very fragile thus perceiving bonds with longer maturity as less risky. A final remark, is that throughout 2012 the interest rates have been consistently decreasing, the current interest rate on bonds of 5 years is 6,73% and 8,62% on bonds with 10 years.