UNIVERSIDADE CATÓLICA PORTUGUESA

Microeconomics I

2st Mini-test   Year 2012/2013

Name: Student number: Class:

The Portuguese market for fixed gear bicycles has so far a single manufacturer, located in Lisbon. The technology used to assemble all the bicycle parts can be represented by $q = \sqrt{2x}$, x being the input used for production, which the firm buys at a constant price $r = 1$.

In the market for these bicycles, the demand is described by $Q = 20 - 2p$.

a) Determine the price, quantity and profit when the monopolist maximizes profit. Represent this equilibrium graphically.

b) If the monopolist is able to implement perfect price discrimination, how much will be produced in equilibrium? Compute the producer surplus and profit, and the consumer surplus.

Suppose now that the firm is considering expanding its activity. The plan is to build another factory in the North. After this expansion, the cost of producing in the Lisbon and Porto factories is given by:

$C_L(q_L) = \frac{q_L^2}{2}$ and $C_P(q_P) = q_P + q_P^2$, respectively.

c) Determine the price, quantities produced in each location and profit when the monopolist maximizes profit.

Solution:

a) The monopolist maximizes $\Pi = \left(\frac{20-q}{2}\right) q - \frac{q^2}{2}$.

$$\frac{\partial \Pi}{\partial q} = 0 \Rightarrow q = 5$$

$$p = \frac{20-5}{2} = 7.5$$

$$\Pi = 7.5 \times 5 - \frac{5^2}{2} = 25$$

Plot: demand, marginal revenue and marginal cost curves. Intercept between MR and MC determines equilibrium quantity, demand the market price for that quantity.

b) 1st degree price discrimination: quantity is the same as in a competitive market.

$$p(q) = MC \Leftrightarrow p(q) = q$$

$$\frac{20-q}{2} = q \Leftrightarrow q = \frac{20}{3}$$

$p = \frac{20}{3}$ (of the 'last unit' sold)

$$\Pi = PS = \int_0^{\frac{20}{3}} p\, dp + \int_{\frac{20}{3}}^{10} (20-2p)\, dp = \frac{100}{3}$$

$$CS = 0$$

(Could also be easily solved geometrically)

c) The monopolist maximizes $\Pi = \left(\frac{20-(q_L+q_P)}{2}\right)(q_L+q_P) - \frac{{q_L}^2}{2} - (q_P + q_P^2)$

$$\begin{cases} \dfrac{\partial \Pi}{\partial q_L} = 0 \\ \dfrac{\partial \Pi}{\partial q_P} = 0 \end{cases} \Leftrightarrow \begin{cases} \dfrac{20-(q_L+q_P)}{2} - \dfrac{q_L+q_P}{2} - q_L = 0 \\ \dfrac{20-(q_L+q_P)}{2} - \dfrac{q_L+q_P}{2} - 1 - 2q_P = 0 \end{cases} \Leftrightarrow \begin{cases} q_L = \dfrac{21}{5} \\ q_P = \dfrac{8}{5} \end{cases}$$

$$p = \frac{20-(q_L+q_P)}{2} = \frac{20-(\frac{21}{5}+\frac{8}{5})}{2} = 7.1$$

$$\Pi = 7.1 \times \left(\frac{21}{5}+\frac{8}{5}\right) - \frac{1}{2}\left(\frac{21}{5}\right)^2 - \left(\frac{8}{5}+\left(\frac{8}{5}\right)^2\right) = 28.2$$