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Microeconomics I

1st Mini-test Year 2012/2013

Name: Student number: Class:

Mateus is a teenager who loves to go to the movie theater on Sundays. To properly enjoy the movies, Mateus also needs to eat popcorns. His utility function is given by $U(x,y) = (x-1)^{0,7}y^{0,3}$, $x \geq 1$, where x is the number of movies, that must be an integer number $x = 1, 2, \dots, N$, and y the weight of popcorn, in grams. Each time Mateus goes to the movie theater, his mother gives him €27. The price of each movie is €7 and the price of each gram of popcorn is €0,03.

a) Determine the optimal choice of Mateus and represent it graphically.
b) Determine the demand functions.

Mateus´ mother imposes that he can only spend 5 hours in the movie theater each Sunday and each movie lasts for 2 hours.

c) Considering all the constraints, what is going to be Mateus new choice? Represent it graphically.

Solutions:

a)

$Max\ U(x,y) = (x-1)^{0,7}y^{0,3}$
$s.t: M = P_x x + P_y y$

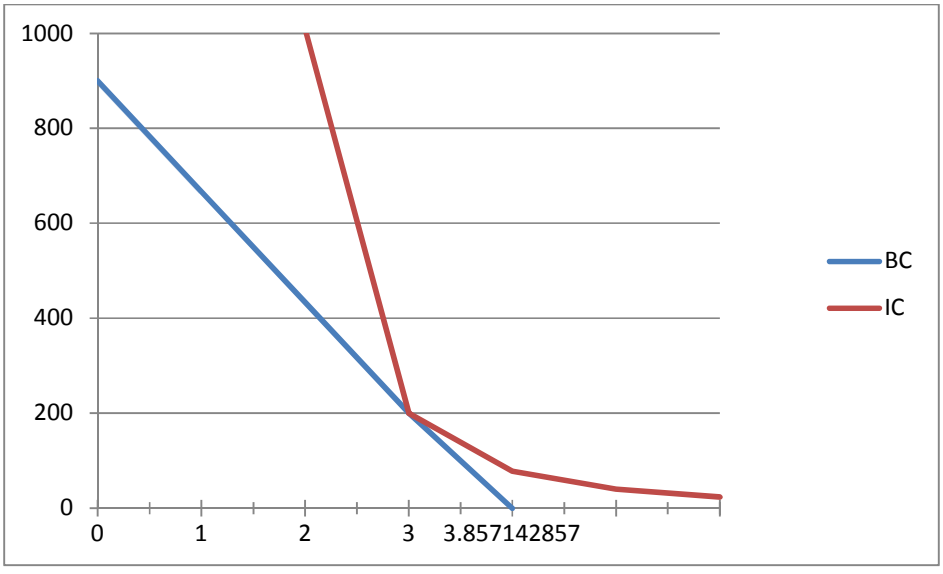
$$\begin{cases} MRS = \frac{P_x}{P_y} \\ M = P_x x + P_y y \end{cases} \Leftrightarrow \begin{cases} \frac{0,7(x-1)^{-0,3}y^{0,3}}{0,3(x-1)^{0,7}y^{-0,7}} = \frac{P_x}{P_y} \\ M = P_x x + P_y y \end{cases} \Leftrightarrow \begin{cases} \frac{7}{3} \times \frac{y}{x-1} = \frac{P_x}{P_y} \\ M = P_x x + P_y y \end{cases} \Leftrightarrow \begin{cases} y = \frac{3}{7} \times \frac{P_x}{P_y} \times (x-1) \\ M = P_x x + P_y \left[\frac{3}{7} \times \frac{P_x}{P_y} \times (x-1)\right] \end{cases} \Leftrightarrow$$

$$\begin{cases} y^* = \frac{3}{7} \times \frac{P_x}{P_y} \times \frac{\frac{7}{10}m + \frac{3}{10}P_x}{P_x} - \frac{3}{7} \times \frac{P_x}{P_y} \\ x^* = \frac{\frac{7}{10}m + \frac{3}{10}P_x}{P_x} \end{cases}$$

Substituting $P_x = 7, P_y = 0{,}03$ and $M = 27$ in the demand function computed above we will have the following optimal solution:

$$\begin{cases} x^* = 3 \\ y^* = 200 \end{cases}$$

Graphically we have:



b) From the previous question we have the following demands functions:

$y^* = \frac{3}{7} \times \frac{\frac{7}{10}m + \frac{3}{10}P_x}{P_y} - \frac{3}{7} \times \frac{P_x}{P_y}$ and $x^* = \frac{\frac{7}{10}m + \frac{3}{10}P_x}{P_x}$.

c) With the time constraint imposed by Matheus´ mother he can only see a maximum of 2,5 movies $\left({}^{5}/_{2} = 2{,}5\right)$. The problem is that the number of movies must be an integer. So, the maximum amount of movies that he can watch is actually 2.
From exercise a) we know that $x^* = 3$ and $y^* = 200$. Now the best way to be as close as possible from this situation is to watch 2 movies and use the remaining money to buy popcorns.

$$\begin{cases} x^* = 2 \\ y^* = \frac{M - P_x \times 2}{P_y} \end{cases} \Leftrightarrow \begin{cases} x^* = 2 \\ y^* = \frac{27 - 7 \times 2}{0.03} \end{cases} \Leftrightarrow \begin{cases} x^* = 2 \\ y^* = 433{,}3(3) \end{cases}$$