Microeconomics I - Midterm

Undergraduate Degree in Business Administration and Economics

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Please answer each group in a separate sheet. Good luck!

Question 1 - Consumers in Fitland have preferences given by

$$u(x, y) = \min(y, 2x),$$

where x is number of hours at the gym, and y is consumption of other goods. Each consumer has an income of $m = 20$ and the price of hours at the gym is $p = 2$, whereas the price of good y is normalized to one.

(a) Find the demand for x and y as a function of m and p. Are the two goods complements or substitutes? Determine the optimal choice for the initial level of prices and income. (2 points)

(b) Consider a price increase to $p = 3$. Decompose it into substitution and income effect. How much would income have to increase to keep the initial utility level? (2 points)

(c) Now suppose the gym offers a membership where consumers pay a fee of 5 to go to the gym at a price of $p = 1$. What would be the optimal choice if consumers chose the new plan? Will consumers prefer the initial price or the new plan? (2 points)

Question 2 - Consider a firm that has a production function given by

$$F(k, l) = 4 + \sqrt{kl}.$$

The price of capital is 8 and the price of labor is 2.

(a) What are returns to scale? How do you characterize the technology above regarding its returns to scale? (2 points)

(b) Determine the optimal choice of capital and labor given the factor prices and determine the long run cost curve. (2 points)

(c) Assuming this firm is a price taker, determine the individual supply curve $q(p)$. (2 points)

(d) Now assume there are 10 identical price taking firms operating in this market, but there is no free entry or exit. Demand is given by $Q = 100 - 5p$. Determine the long run equilibrium in this market. Will firms be able to make positive profits? (2 points)

Question 3 - Consider a monopolistic market with demand given by

$$Q = 12 - p.$$

The monopolist has two factories where he can produce, which have cost functions given by

$$C_1(q_1) = 4q_1 \qquad C_2(q_2) = 1 + q_2^2.$$

(a) What is an elasticity and how is it calculated? For the demand function above, what is the price elasticity of demand? (2 points)

(b) What are economies of scale? Do any of the cost functions above exhibit economies of scale? (2 points)

(c) Determine the optimal amount produced by the monopolist in each factory. (2 points)

Microeconomics I - Midterm 2013 Solution

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Question 1 - Consumers in Fitland have preferences given by

$$u(x, y) = \min(y, 2x),$$

where x is number of hours at the gym, and y is consumption of other goods. Each consumer has an income of $m = 20$ and the price of hours at the gym is $p = 2$, whereas the price of good y is normalized to one.

(a) Find the demand for x and y as a function of m and p. Are the two goods complements or substitutes? Determine the optimal choice for the initial level of prices and income. (2 points)

Solution: The optimal choice is characterized by the following conditions

$$y = 2x \qquad m = px + y.$$

Solving for x and y, we find the demand functions

$$x = \frac{m}{2+p} \qquad y = \frac{2m}{2+p}.$$

The two goods are complements since the demand for good y decreases when the price of good x increases.

For the initial prices and income, the demand for the two goods is $x = 5$ and $y = 10$.

(b) Consider a price increase to $p = 3$. Decompose it into substitution and income effect. How much would income have to increase to keep the initial utility level? (2 points)

Solution: At a price of $p = 3$, the final bundle consumed is $x = 4$ and $y = 8$.

The initial utility was $u = 10$. To find the substitution effect we need to find the bundle that attains this utility, and that is the optimal choice at the new prices. In order to attain a utility of $u = 10$, the consumer will need to consume $x = 5$ and $y = 10$, which means that there is no substitution effect. To purchase this bundle at the new prices, the consumer would need an income of $m = 25$.

The income effect results from the decrease in income from 25 to 20 (at the new prices) and leads to a decrease in x from 5 to 4, and a decrease in y from 10 to 8.

(c) Now suppose the gym offers a membership where consumers pay a fee of 5 to go to the gym at a price of $p = 1$. What would be the optimal choice if consumers chose the new plan? Will consumers prefer the initial price or the new plan? (2 points)

Solution: With the new membership the available income for the consumer is reduced to $m = 15$, and the price of good x becomes $p = 1$. For these parameters, the demand for the two goods is $x = 5$ and $y = 10$, which leads to the same utility as the initial price, which means the consumer is indifferent between subscribing to the membership or not.

Question 2 - Consider a firm that has a production function given by

$$F(k, l) = 4 + \sqrt{kl}.$$

The price of capital is 8 and the price of labor is 2.

(a) What are returns to scale? How do you characterize the technology above regarding its returns to scale? (2 points)

Solution: Returns to scale refer to the percentage change in output when all factors of production increase in the same proportion. If the percentage change in output is higher than in the inputs we have increasing returns to scale, if it is lower we have decreasing returns to scale, and if it is the same we have constant returns to scale. In order to evaluate the returns to scale in this production function, let's see what happens to output when we multiply the amount of each output by a factor of $\lambda > 1$

$$F(\lambda k, \lambda l) = 4 + \lambda\sqrt{kl} < 4\lambda + \lambda\sqrt{kl} = \lambda F(k, l).$$

Since output increases by less than a factor of λ, we have decreasing returns to scale.

(b) Determine the optimal choice of capital and labor given the factor prices and determine the long run cost curve. (2 points)

Solution: To produce any quantity lower than 4, no factors of production need to be used, so the optimal choice is $k = l = 0$.

If $q \geq 4$, the firms solve the usual cost minimizing problem. In the optimal solution, the ratio of factor prices equal the MRTS and output is given by the production function

$$MRTS = \frac{l}{k} = \frac{8}{2} \rightarrow l = 4k, \;\; q = 4 + \sqrt{kl} = 4 + 2k \rightarrow k = \frac{1}{2}(q-4), \;\; l = 2(q-4).$$

The cost function is thus given by

$$C(q) = \begin{cases} 0 & \text{for } \; q < 4 \\ 8(q-4)/2 + 2 * 2(q-4) = 8(q-4) & \text{for } \; q \geq 4. \end{cases}$$

(c) Assuming this firm is a price taker, determine the individual supply curve $q(p)$. (2 points)

Solution: For this cost function, the marginal cost is zero for $q < 4$ and 8 for $q > 4$. A price taking firm chooses the quantity where prices equal marginal costs. Individual supply is thus

$$q(p) = \begin{cases} \text{any } \; q < 4 & \text{for } \; p = 0 \\ q = 4 & \text{for } \; 0 \leq p \leq 8 \\ \text{any } \; q > 4 & \text{for } \; p = 8. \end{cases}$$

(d) Now assume there are 10 identical price taking firms operating in this market, but there is no free entry or exit. Demand is given by $Q = 100 - 5p$. Determine the long run equilibrium in this market. Will firms be able to make positive profits? (2 points)

Solution: For $p = 0$, aggregate supply is lower than 40, and aggregate demand is 100, so there is no equilibrium.

For $0 < p < 8$, aggregate supply is 40, and aggregate demand is between 60 and 100, so there is no equilibrium.

For $p = 8$, aggregate supply is higher than 40, and aggregate demand is 60, so we have an equilibrium where each individual firm is producing at least 4 units, and their total production is 60.

Each firm's profits are given by $\pi = 8q - 8(q-4) = 32$. Firms are able to make positive profits because there is no free entry.

Question 3 - Consider a monopolistic market with demand given by

$$Q = 12 - p.$$

The monopolist has two factories where he can produce, which have cost functions given by

$$C_1(q_1) = 4q_1 \qquad C_2(q_2) = 1 + q_2^2.$$

(a) What is an elasticity and how is it calculated? For the demand function above, what is the price elasticity of demand? (2 points)

Solution: The elasticity of x with respect to y gives us the percentage variation in x when y increases 1%. It can be calculated using one of the following expressions

$$\varepsilon_{xy} = \frac{\frac{dx}{x}}{\frac{dy}{y}} = \frac{dx}{dy}\frac{y}{x} = \frac{d\ln x}{d\ln y}.$$

In this case, the price elasticity of demand is the percentage variation in the quantity demanded when the price increases 1%, and it is given by

$$\varepsilon_{Qp} == \frac{dQ}{dp}\frac{p}{Q} = -\frac{p}{12-p}.$$

(b) What are economies of scale? Do any of the cost functions above exhibit economies of scale? (2 points)

Solution: There are economies of scale whenever AVERAGE costs are decreasing with the quantity produced. For the cost functions above, the average cost curves are given by

$$AC_1(q_1) = 4 \qquad AC_2(q_2) = \frac{1}{q_2} + q_2.$$

In factory 1 average costs are constant, so there are no economies or diseconomies of scale. In factory 2 there are economies of scale if the following condition is met

$$\frac{dAC_2}{dq_2} = -\frac{1}{q_2^2} + 1 < 0 \rightarrow q_2 < 1.$$

For $q_2 > 1$, there are diseconomies of scale since average costs are increasing with quantity.

(c) Determine the optimal amount produced by the monopolist in each factory. (2 points)

Solution: If the monopolists produces a positive amount in both factories, it solves the following problem

$$\max \pi = (q_1 + q_2)(12 - q_1 - q_2) - 4q_1 - 1 - q_2^2.$$

The first order conditions for this problem are

$$\begin{aligned}
\frac{d\pi}{dq_1} &= 0 \rightarrow 12 - 2q_1 - 2q_2 = 4 \\
\frac{d\pi}{dq_2} &= 0 \rightarrow 12 - 2q_1 - 2q_2 = 2q_2.
\end{aligned}$$

Solving the two equation, we find $q_2 = 2$ and $q_1 = 2$, which leads to $p = 8$. Total profits are $\pi = 4 * 8 - 4 * 2 - 1 - 2^2 = 32 - 13 = 19$.

The firm also has the option of only producing in one of the factories. If it only produces in factory 1, the optimal quantity is $q_1 = 4$, which leads to $p = 8$ and $\pi = 4*8-4*4 = 16$. If it only produces in factory 2, the optimal quantity is $q_1 = 3$, which leads to $p = 9$ and $\pi = 3 * 9 - 1 - 3^2 = 27 - 10 = 17$.

Since profits are higher when the monopolist uses both factories, that will be the chosen solution.