CATÓLICA LISBON SCHOOL OF BUSINESS & ECONOMICS	**Matemática I** Resolução do 1ºMini-Teste **1 de Março de 2011**

Grupo I

1) Calcule o determinante: $\begin{vmatrix} 3 & 5 & 1 & 2 & 4 \\ 2 & 1 & 1 & 3 & 0 \\ 4 & 2 & 0 & 1 & 3 \\ 5 & 2 & 2 & 2 & 4 \\ 2 & 1 & 3 & 0 & 5 \end{vmatrix}$. **(3.5)**

$$\begin{vmatrix} 3 & 5 & 1 & 2 & 4 \\ 2 & 1 & 1 & 3 & 0 \\ 4 & 2 & 0 & 1 & 3 \\ 5 & 2 & 2 & 2 & 4 \\ 2 & 1 & 3 & 0 & 5 \end{vmatrix} \begin{matrix} \\ L_2 - L_1 \\ \\ L_4 - 2L_1 \\ L_5 - 3L_1 \end{matrix} = \begin{vmatrix} 3 & 5 & 1 & 2 & 4 \\ -1 & -4 & 0 & 1 & -4 \\ 4 & 2 & 0 & 1 & 3 \\ -1 & -8 & 0 & -2 & -4 \\ -7 & -14 & 0 & -6 & -7 \end{vmatrix} = 1(-1)^{1+3} \begin{vmatrix} -1 & -4 & 1 & -4 \\ 4 & 2 & 1 & 3 \\ -1 & -8 & -2 & -4 \\ -7 & -14 & -6 & -7 \end{vmatrix} =$$

$$C_2 \to C_2 - C_4$$

$$\begin{vmatrix} -1 & 0 & 1 & -4 \\ 4 & -1 & 1 & 3 \\ -1 & -4 & -2 & -4 \\ -7 & -7 & -6 & -7 \end{vmatrix} = \begin{vmatrix} -1 & 0 & 0 & 0 \\ 4 & -1 & 5 & -13 \\ -1 & -4 & -3 & 0 \\ -7 & -7 & -13 & 21 \end{vmatrix} = (-1)(-1)^{1+1} \begin{vmatrix} -1 & 5 & -13 \\ -4 & -3 & 0 \\ -7 & -13 & 21 \end{vmatrix} = -(63 - 676 + 273 + 420)$$

$$C_3 \to C_3 + C_1$$
$$C_4 \to C_4 - 4C_1$$

$$= -80$$

2) Resolva a equação seguinte: $\begin{vmatrix} 6x^2 - 4x & 2x-2 & 3x+6 \\ x^2 + x & x-1 & x+2 \\ 6x^2+2x & 4x-4 & 3x+6 \end{vmatrix} = 0$. **(4.0)**

$$\begin{vmatrix} 6x^2 - 4x & 2x-2 & 3x+6 \\ x^2 + x & x-1 & x+2 \\ 6x^2+2x & 4x-4 & 3x+6 \end{vmatrix} = \begin{vmatrix} 2x(3x-2) & 2(x-1) & 3(x+2) \\ x(x+1) & x-1 & x+2 \\ 2x(3x+1) & 4(x-1) & 3(x+2) \end{vmatrix} = x(x-1)(x+2) \begin{vmatrix} 6x-4 & 2 & 3 \\ x+1 & 1 & 1 \\ 6x+2 & 4 & 3 \end{vmatrix} =$$

$$C_1 \to C_1 : x \qquad C_3 \to C_3 - C_2$$
$$C_2 \to C_2 : (x-1)$$
$$C_3 \to C_3 : (x+2)$$

$$= x(x-1)(x+2)\begin{vmatrix} 6x-4 & 2 & 1 \\ x+1 & 1 & 0 \\ 6x+2 & 4 & -1 \end{vmatrix} \begin{matrix} \\ \\ L_3+L_1 \end{matrix} = x(x-1)(x+2)\begin{vmatrix} 6x-4 & 2 & 1 \\ x+1 & 1 & 0 \\ 12x-2 & 6 & 0 \end{vmatrix} = x(x-1)(x+2)\begin{vmatrix} x+1 & 1 \\ 12x-2 & 6 \end{vmatrix} =$$

$$= x(x-1)(x+2)(-6x+8)$$

$$x(x-1)(x+2)(-6x+8) = 0 \Leftrightarrow x = 0 \vee x = 1 \vee x = -2 \vee x = \frac{4}{3}$$

3) Calcule o determinante, apresentando o resultado sob a forma de um produto de factores do 1º grau.

$$\begin{vmatrix} a & b & a & b \\ 2a & 2a-b & 2a+b & a+3b \\ -a & -2b & 3b & 2a-4b \\ a & -3a-6b & 3a+5b & 4a+2b \end{vmatrix}$$

(4.0)

$$\begin{vmatrix} a & b & a & b \\ 2a & 2a-b & 2a+b & a+3b \\ -a & -2b & 3b & 2a-4b \\ a & -3a-6b & 3a+5b & 4a+2b \end{vmatrix} \begin{matrix} = \\ L_2-2L_1 \\ L_3+L_1 \\ L_4-L_1 \end{matrix} \begin{vmatrix} a & b & a & b \\ 0 & 2a-3b & b & a+b \\ 0 & -b & 3b+a & 2a-3b \\ 0 & -3a-7b & 2a+5b & 4a+b \end{vmatrix} = a\begin{vmatrix} 2a-3b & b & a+b \\ -b & 3b+a & 2a-3b \\ -3a-7b & 2a+5b & 4a+b \end{vmatrix} =$$

$$C_1 \to C_1 + C_2 + C_3$$

$$= a\begin{vmatrix} 3a-b & b & a+b \\ 3a-b & 3b+a & 2a-3b \\ 3a-b & 2a+5b & 4a+b \end{vmatrix} \begin{matrix} = \\ L_2-L_1 \\ L_3-L_1 \end{matrix} \quad a\begin{vmatrix} 3a-b & b & a+b \\ 0 & 2b+a & a-4b \\ 0 & 2a+4b & 3a \end{vmatrix} = a(3a-b)\begin{vmatrix} 2b+a & a-4b \\ 2(2b+a) & 3a \end{vmatrix} =$$

$$C_1 \to C_1 : (2b+a)$$

$$= a(3a-b)(2b+a)\begin{vmatrix} 1 & a-4b \\ 2 & 3a \end{vmatrix} = a(3a-b)(2b+a)(a+8b)$$

Grupo II

4) Calcule a inversa da matriz $A=\begin{bmatrix}2&4&3\\0&1&1\\2&2&-1\end{bmatrix}$**.** **(2.5)**

$$A^{-1}=\frac{\begin{bmatrix}-3&2&-2\\10&-8&4\\1&-2&2\end{bmatrix}^T}{-4}=\frac{\begin{bmatrix}-3&10&1\\2&-8&-2\\-2&4&2\end{bmatrix}}{-4}=\begin{bmatrix}\frac{3}{4}&-\frac{5}{2}&-\frac{1}{4}\\-\frac{1}{2}&2&\frac{1}{2}\\\frac{1}{2}&-1&-\frac{1}{2}\end{bmatrix}$$

5) Supondo que A e B **são matrizes regulares, resolva a equação matricial em ordem a** X :

$$\left\{\left[(A^T)^{-1}X\right]^T+(AB)^{-1}\right\}^{-1}=A \quad \textbf{(3.0)}$$

$$\left[(A^T)^{-1}X\right]^T+(AB)^{-1}=A^{-1}\Leftrightarrow X^TA^{-1}+B^{-1}\mathrm{A}^{-1}=A^{-1}\Leftrightarrow X^TA^{-1}=A^{-1}-B^{-1}\mathrm{A}^{-1}\Leftrightarrow$$

$$\Leftrightarrow X^TA^{-1}=(I-B^{-1})\mathrm{A}^{-1}\Leftrightarrow X^TA^{-1}A=(I-B^{-1})\mathrm{A}^{-1}A\Leftrightarrow X^T=(I-B^{-1})\Leftrightarrow X=(I-B^{-1})^T$$

6) Determine a expressão das matrizes permutáveis com a matriz $A=\begin{bmatrix}1&2\\-1&-1\end{bmatrix}$**.** **(3.0)**

$$AB=BA\Leftrightarrow\begin{bmatrix}1&2\\-1&-1\end{bmatrix}\begin{bmatrix}a&b\\c&d\end{bmatrix}=\begin{bmatrix}a&b\\c&d\end{bmatrix}\begin{bmatrix}1&2\\-1&-1\end{bmatrix}\Leftrightarrow\begin{bmatrix}a+2c&b+2d\\-a-c&-b-d\end{bmatrix}=\begin{bmatrix}a-b&2a-b\\c-d&2c-d\end{bmatrix}\Leftrightarrow$$

$$\begin{cases}a+2c=a-b\\-a-c=c-d\\b+2d=2a-b\\-b-d=2c-d\end{cases}\Leftrightarrow\begin{cases}b=-2c\\d=a+2c\\d=a-b\\b=-2c\end{cases}\Leftrightarrow\begin{cases}b=-2c\\d=a+2c\\d=d\\b=b\end{cases},\forall_{a,c\in\mathbb{R}}\qquad A=\begin{bmatrix}a&-2c\\c&a+2c\end{bmatrix},\forall_{a,c\in\mathbb{R}}$$