## Group I

Mr Smith is a trader of forward contracts on copper. Currently, he holds the following contracts on copper:

- 5 long positions, in forward contract for delivery in 3 months, with a delivery price K=6; this position was opened 2 months ago. The current market value of this position is 7,9.
- 7 long positions, in forward contract for delivery in 5 months, with a delivery price K=7; this position was opened 1 month ago.
- 4 short positions, in forward contract for delivery in 5 months; this position was opened 1 month ago.

Additional assumptions:

- The current market price of a ton of copper is 8;
- Each contract is on 1 ton of copper;
- The net convenience yield on copper is constant;
- The continuously compounded risk-free interest rate is constant and equal to 4%.

1. What has been happening to the price of copper in the spot market for the last 2 months?
2. What is currently the forward price of copper, for delivery in 3 months?
3. What is the net convenience yield on copper?
4. What is currently the forward price of copper, for delivery in 5 months?
5. What should be the current market value of the overall position in forward contracts on copper held by Mr Smith?
6. Plot the payoff of the overall position of Mr. Smith, in 3 months, as a function of the realized value of the spot price of copper (i.e., the spot price of copper 3 months from today). Be detailed as possible in your plot, giving all required numerical information to fully characterize the plot.
7. What additional position should Mr Smith take today in forward contracts on copper to fully eliminate his exposure to copper price risk?
8. What annual-equivalent rate of return should Mr Smith expect to earn on his overall position in forward contracts over the next 3 months, after taking the additional position indicated in the previous question?

**Group II**

We have collected the following information on options on the stock of Carlsberg (each option is on 1 share of Carlsberg; the stock is not expected to pay dividends over the next 3 months):

| | European call – 3 months to maturity | | | | European put – 3 months to maturity | | | |
|---|---|---|---|---|---|---|---|---|
| Strikes | Current price | Intrinsic value | Time value | Lower limit | Current price | Intrinsic value | Time value | Lower limit |
| 7 | 3,134 | | | | | | | |
| 10 | | | | | | | 0,977 | |
| 13 | 0,254 | | | | | | 0,18 | |

One share of Carlsberg stock trades currently at 10 euros; the annual risk-free rate with continuously compounding is 5%.

1. Fill in all the blank cells in the Table;
2. Do you see any arbitrage opportunity? Which one exactly?
3. If you found an arbitrage opportunity in the previous question, explain how you would set up an investment strategy to profit from it.

A trader has the following portfolio of 3-months European options on Carlsberg stock:

- 10 short positions in puts (X=7);
- 18 long positions in calls (X=7);
- 12 short position is calls (X=10);
- 4 long positions in puts (X=10).

4. Plot the payoff of the position of the trader at the maturity date of the options, as a function of the share price of Carlsberg, ignoring any initial cash-flow required to set up the position. Be detailed as possible in your plot, giving all required numerical information to fully characterize the plot.
5. What <u>downside</u> risk is the trader exposed to?

## Group III

The stock of BMW, currently trading at 70 euros per share, has an historical volatility of 35% (standard deviation on annual returns, based on continuous compounding). The company announced its intention to pay a dividend per share of 5 euros in 6 months. The annual risk-free interest rate, with continuous compounding, is 4%.

1.Draw a binomial tree for the trajectory of stock price of BMW (one share) over the next 12 months, assuming 6-months time increments. Make sure to write the value of the stock price at each node of the tree.

2. Draw the binomial tree of an at-the-money American call option on one share of BMW stock, with 1-year to maturity, assuming 6-months time increments. Make sure to write down the value of the option and the delta of the option, at each node of the tree.

3. Draw the binomial tree of an at-the-money European call option on one share of BMW stock, with 1-year to maturity, assuming 6-months time increments. Make sure to write the value of the option and the delta of the option, at each node of the tree.

4. Suppose the market price of the American call exceeded the price computed in question 2. Would that represent an arbitrage opportunity? Yes or no? Explain your answer.

5. Again, suppose the market price of the American call exceeded the price computed in question 2. You, however, believe strongly that the binomial model used to estimate the value of the option correctly prices it. If the model is indeed correct, what investment strategy would allow you to exploit the deviation between the market price and the estimated price of the option? Be detailed in your answer.

## Group IV

Bank DELTA is considering selling a short-term structured product to retail investors through its branch network. The bank is expected to keep the position resulting from the issuance of the product in its books until the maturity of the product.

You have the following information about the product:

Maturity: 2 weeks
Selling price, per unit of product: 1000 euros
Nº of units of product to be sold to customers: 1 million
Payoff at maturity, per unit of product:
1050 euros - 1000 euros * k * R if $R > 0$;
1050 euros - 1000 euros * k * (-R) if $R < 0$

where R represents the (2-week) return on shares of Galp during the life of the structured product (the company is not expected to pay any dividend over the next 2 weeks).

A share of Galp is currently worth 10 euros. The annual volatility of returns on Galp shares (based on continuous compounding) is estimated to be equal to 50%. The risk-free interest rate (based on continuous compounding) is 4%.

1. Characterize one unit of the structured product in terms of its building blocks;

    (a) Deposit of PV(1050); long 1000k 2-weeks at-the-money calls; long 1000k 2-weeks at-the-money puts;
    (b) Deposit of PV(1050); short 1000k 2-weeks at-the-money calls; long 1000k 2-weeks at-the-money puts;
    (c) Deposit of PV(1050); long 1000k 2-weeks at-the-money calls; short 1000k 2-weeks at-the-money puts;
    (d) Deposit of PV(1050); short 1000k 2-weeks at-the-money calls; short 1000k 2-weeks at-the-money puts;
    (e) Deposit of PV(1050); long 100k 2-weeks at-the-money calls; long 100k 2-weeks at-the-money puts;
    (f) Deposit of PV(1050); short 100k 2-weeks at-the-money calls; long 100k 2-weeks at-the-money puts;
    (g) Deposit of PV(1050); long 100k 2-weeks at-the-money calls; short 100k 2-weeks at-the-money puts;
    (h) Deposit of PV(1050); short 100k 2-weeks at-the-money calls; short 100k 2-weeks at-the-money puts;

2. Draw a two-week binomial tree with weekly increments for the value of a share of Galp;

3. Using a binomial model with weekly increments, determine what should be the value of k to guarantee bank DELTA a profit margin of 5% at the date of issuance of the structured product (i.e., theorectical cost of product is 95% of its selling price).

4. Plot the payoff of one unit of the structured product at maturity date, as a function of the value of a share of Galp. Make sure to show in the plot all the numerical information required to fully characterize the payoff;

5. Using a binomial tree with weekly increments, show how the position of bank DELTA resulting from the issuance of the structured product might evolve during the life of the product. At each node of the tree, indicate (i) the value of the position held by the bank and (ii) the delta of the position held by the bank.

6. The position of bank DELTA resulting from the issuance of the structured product has:
   (a) a positive gama;
   (b) a negative gama;
   (c) a gama equal to zero;
   (d) you cannot tell the sign of the gamma.

7. The position of bank DELTA resulting from the issuance of the structured product has:
   (a) a positive teta;
   (b) a negative teta;
   (c) a teta equal to zero;
   (d) it is impossible to tell the sign of the teta..

8. Suppose that the bank DELTA delta-hedges the exposure of the position resulting from the issuance of the structured product, using shares of Galp. What should happen to the value of the delta-hedged position during the first week of the product's life, <u>if the value of Galp's shares remains unchanged</u>?
   (a) Should go up;
   (b) Should go down;
   (c) Should remain unchanged;
   (d) It is impossible to tell how it would change.

9. Suppose that the bank DELTA delta-hedges the exposure of the position resulting from the issuance of the structured product, using shares of Galp. What should happen to the value of the delta-hedged position during the first week of the product's life, <u>if the value of Galp's shares experiences a big change</u>?
   (a) Should go up;
   (b) Should go down;
   (c) Should remain unchanged;
   (d) It is impossible to tell how it would change.

10. Simulate the delta hedging of the structured product by Bank DELTA during the life of the product, assuming (i) the bank hedges its position using shares of Galp; (ii) the hedge is readjusted weekly; and (iii) the shares of GALP rise for two consecutive weeks along the binomial tree represented in question 2. Make sure to show the weekly evolution of (at least):
    a. The delta of the (unhedged) position of the bank;

b. The size of the position in shares of Galp required to delta-hedge the bank's exposure;
c. The value of the unhedged position held by the bank;
d. The value of the hedged positions held by the bank.

# GROUP I

① $S_0 = 8$

$S_{-2\,\text{months}} = 6$

$S_{-1\,\text{month}} = 7$

THE PRICE OF COPPER HAS BEEN RISING OVER PAST 2 MONTHS

② $$5\left[F(0,3\,\text{months}) - 6\right] e^{-0{,}04\frac{3}{12}} = 7{,}9$$

$$F(0,3\,\text{months}) = 7{,}596$$

③ $$F(0,3\,\text{months}) = S_0\, e^{(0{,}04 - hcy)\frac{3}{12}}$$

$$7{,}596 = 8\, e^{(0{,}04 - hcy)\frac{3}{12}}$$

$$hcy = 24{,}73\%$$

④ $$F(0,5\,\text{months}) = S_0\, e^{(0{,}04 - 0{,}2473)\frac{5}{12}}$$

$$= 8\, e^{(0{,}04 - 0{,}2473)\frac{5}{12}} = 7{,}34$$

⑤ VALUE OF POSITION IN 5-MONTHS CONTRACTS

$$(7-4)\left\{\underbrace{F(0,5\,\text{months})}_{7{,}34} - 7\right\} e^{-0{,}04\frac{5}{12}} = 1$$

VALUE OF OVERALL POSITION $= 7{,}9 + 1 = 8{,}9$

(6) Payoff of Position in 3 months

$$5\left[S_{3\,months}-6\right]+(7-4)\left[F(3\,months,5\,months)-7\right]e^{-0{,}04\frac{2}{12}}$$

Substituting

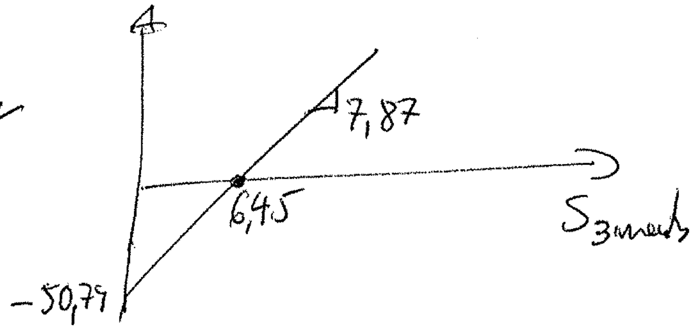
$$F(3\,months,5\,months)=S_{3\,months}\,e^{(0{,}04-0{,}2473)\frac{2}{12}}$$

$$=0{,}966\cdot S_{3\,months}$$

$$5\left[S_{3\,months}-6\right]+3\left[0{,}966\cdot S_{3\,months}-7\right]0{,}99$$

$\Leftrightarrow$

$$7{,}87\,S_{3\,months}-50{,}79$$




(7) Position held by Mr Smith benefits if price of copper rises steadily over next 5 months

(8) • Open 5 short positions in forward contracts maturing in 3 months

• Open 3 short positions in forward contracts maturing in 5 months

(9) The risk-free rate of return (4%)

## Group II



(1) $S=10$

| | Calls – 3 months | | | | Puts – 3 months | | | |
|---|---|---|---|---|---|---|---|---|
| Strikes | Price | Intrinsic Val. | Time Val. | Lower Limit | Price | Intrin. Val. | Time Val. | Lower Limit |
| 7 | 3,134 | 3 | 0,134 | 3,09 | 0,047 | 0 | 0,047 | 0 |
| 10 | 1,1 | 0 | 1,1 | 0,124 | 0,977 | 0 | 0,977 | 0 |
| 13 | 0,254 | 0 | 0,254 | 0 | 3,18 | 3 | 0,18 | 2,84 |

Put-Call Parity:

$$c_{X=10} = S - PV(X=10) + p_{X=10}$$

$$c_{X=10} = 10 - 10\,e^{-0{,}05\frac{3}{12}} + 0{,}977 = 1{,}101$$

$$p_{X=7} = c_{X=7} - S + PV(X=7)$$

$$= 3{,}134 - 10 + 7\,e^{-0{,}05\frac{3}{12}} = 0{,}047$$

Lower Limits:

$c_{X=7}$: $\max\{0,\ S - PV(X=7)\} = \max\{0,\ 10 - 7e^{-0{,}05\frac{3}{12}}\}$

$= \max\{0,\ 3{,}09\} = 3{,}09$

$c_{X=10}$: $\max\{0,\ 10 - 10e^{-0{,}05\frac{3}{12}}\} = \max\{0,\ 0{,}124\} = 0{,}124$

$c_{X=13}$: $\max\{0,\ 10 - 13e^{-0{,}05\frac{3}{12}}\} = 0$

$p_{X=7}$: $\max\{0;\ 7e^{-0{,}05\frac{3}{12}} - 10\} = 0$

$p_{X=10}$: $\max\{0;\ 10e^{-0{,}05\frac{3}{12}} - 10\} = 0$

$p_{X=13}$: $\max\{0,\ 13e^{-0{,}05\frac{3}{12}} - 10\} = 2{,}84$

(2) ARBITRAGE OPPORTUNITY

$C_{X=13}$ & $p_{X=13}$ ARE INCONSISTENT with put-call PARITY

$C_{X=13}$ ACCORDING TO put-call parity SHOULD BE WORTH:

$$= S - PV(X=13) + p_{X=13}$$

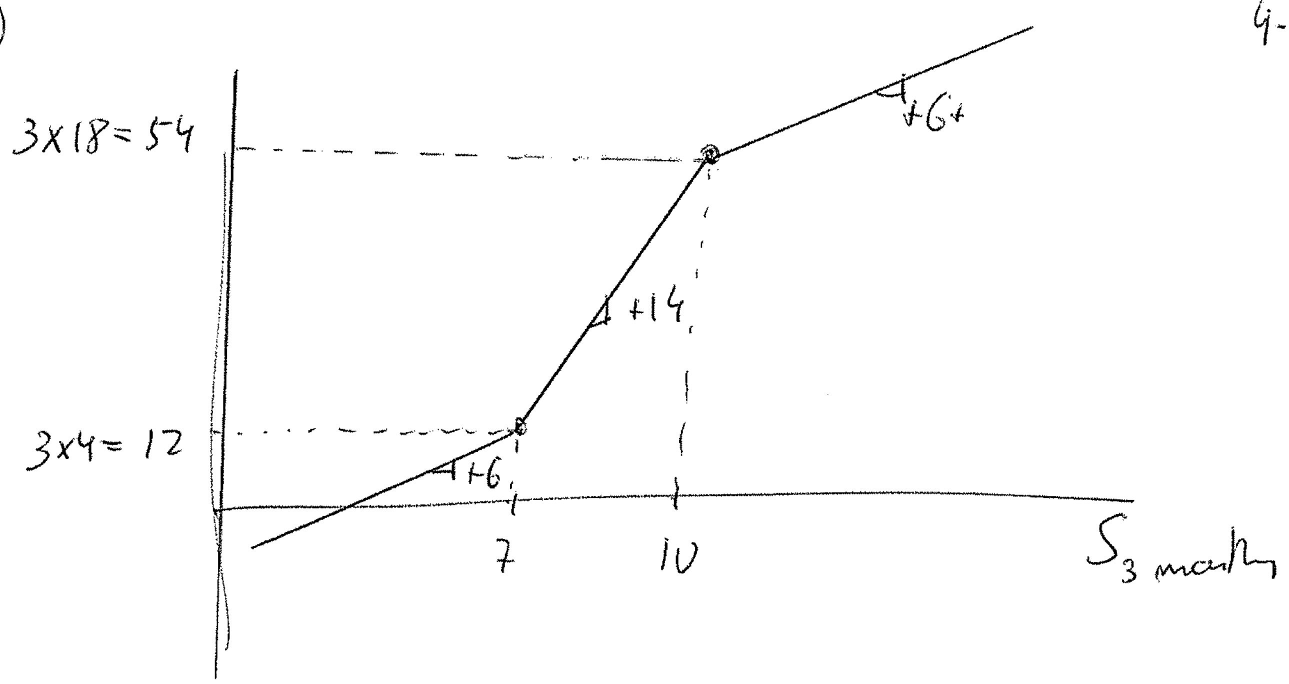
$$= 10 - 13\,e^{-0{,}05\frac{3}{12}} + 3{,}18 = 0{,}3412$$

INVESTMENT STRATEGY

| | CASH-FLOWS DATE $\phi$ | DATE T (3 months) |
|---|---|---|
| • Buy call | $-0{,}254$ | |
| • Sell Stock | $+10$ | |
| • Sell Put | $+3{,}18$ | |
| • Deposit $PV(X=13)$ | $-12{,}84$ | |
| | $+0{,}086$ | $\phi$ |

③





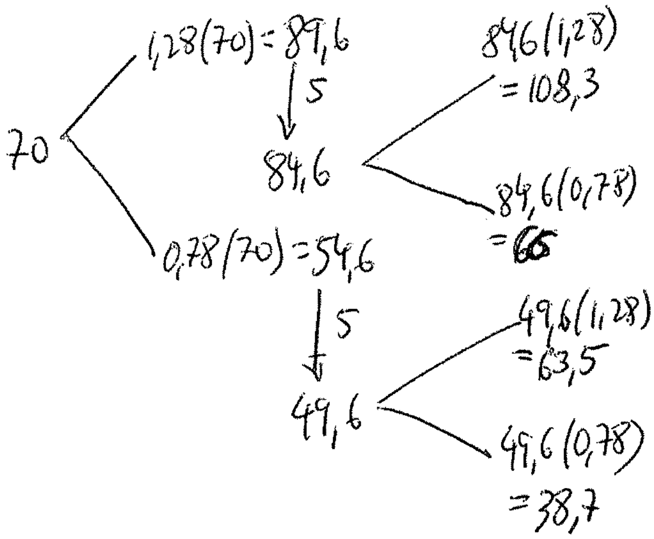

④ Portfolio loses money if $S_\star$

# Group III

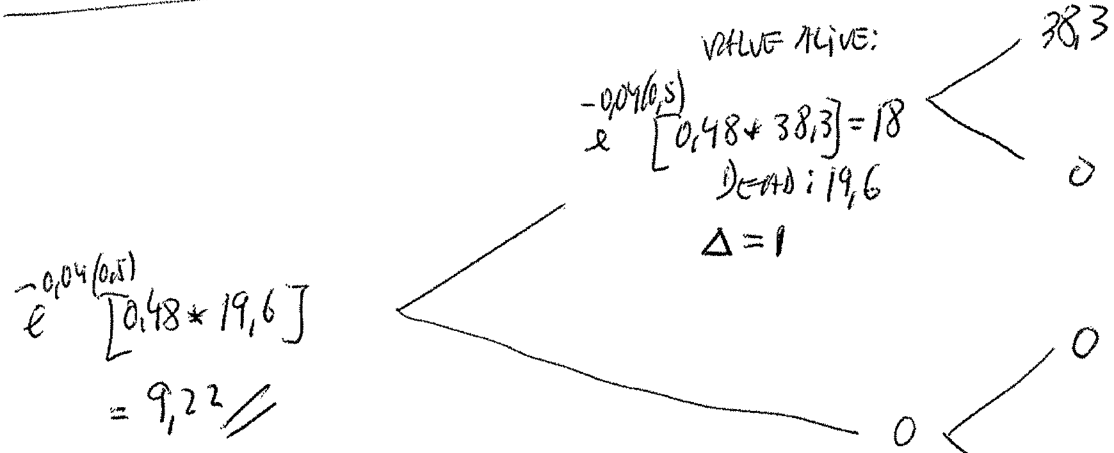
$\sigma = 0{,}35$, $\Delta t = 0{,}5$

$$u = e^{\sigma\sqrt{\Delta t}} = e^{0{,}35\sqrt{0{,}5}} = 1{,}28 \qquad d = \frac{1}{u} = 0{,}78$$

$$\pi = \frac{e^{0{,}04(0{,}5)} - 0{,}78}{1{,}28 - 0{,}78} = 48\% \qquad 1-\pi = 52\%$$




## American Call (X = 70):



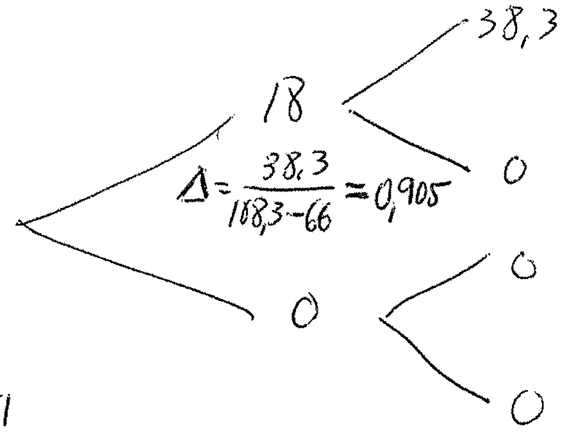

$$\Delta = \frac{19{,}6 - 0}{89{,}6 - 54{,}6} = 0{,}56$$

VALUE OF AMERICAN CALL = $100 \times 9{,}22 = 922$ €

② EUROPEAN CALL ($X=70$)

$$e^{-0{,}04(0{,}5)}\left[0{,}48 * 18\right] = 8{,}47$$

$$\Delta = \frac{18}{89{,}6-54{,}6} = 0{,}51$$




VALUE OF EUROPEAN CALL $= 100 \times 8{,}47 = 847$

③ THERE WOULD BE NO ARBITRAGE OPPORTUNITY; ESTIMATE OF VALUE OF CALL IS A GOOD AS THE MODEL USED TO COMPUTE IT.

④ SELL CALL
BUY SYNTHETIC CALL

- Buy $0{,}56 \times 100 = 56$ shares of BOY
- Borrow $\left[\Delta_0 S_1^u - C_1^u\right] e^{-r\Delta t} = \left[0{,}56 * 89{,}6 - 19{,}6\right] e^{-0{,}04(0{,}5)} * 100$
  $= 2997$ Euros

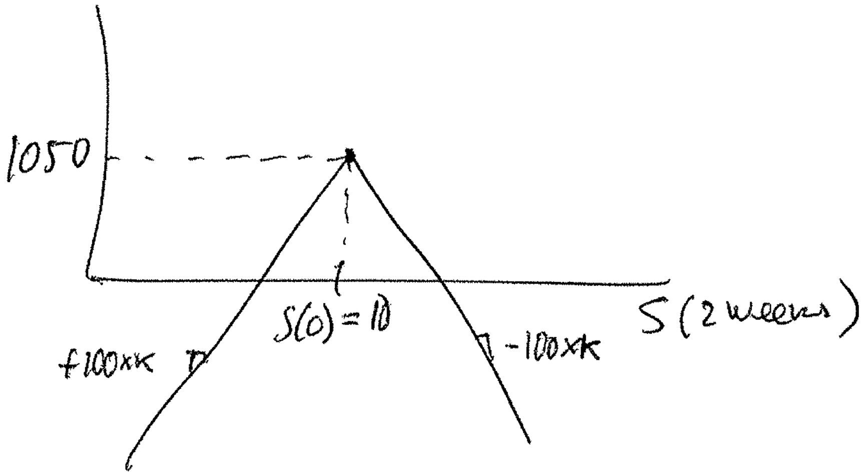
6 months later
if $S_1^d$: VALUE SYNTHETIC CALL $= 0$ (= REAL CALL)
if $S_1^u$: VALUE of SYNTHETIC CALL $= 19{,}6$ (= REAL CALL)

# GROUP IV



## GRP

Payoff of structured product at maturity




Structured product consists of:

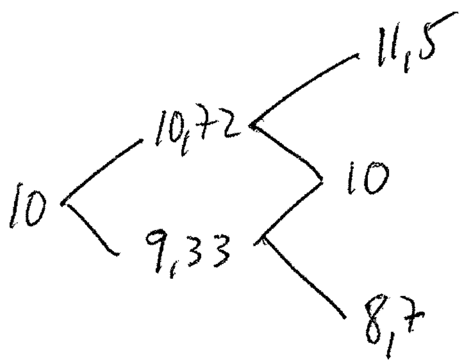
Deposit of $PV(1050) = 1050\, e^{-0{,}04\frac{2}{52}} = 1048{,}4$

100xk short at-the-money calls

100xk short at-the-money puts

Binomial tree for GRP: $u = e^{0{,}5\sqrt{\frac{1}{52}}} = 1{,}072$

$d = (1{,}072)^{-1} = 0{,}933$




$$\pi = \frac{e^{0{,}04\frac{1}{52}} - 0{,}933}{1{,}072 - 0{,}933} = 0{,}488$$

$1 - \pi = 0{,}512$

Value of at-the-money call:

$$e^{-0{,}04\left(\frac{1}{52}\right)}(0{,}488)(1{,}5) = 0{,}731$$

$$\delta_c = \frac{1{,}5}{1{,}5} = 1$$

$$1{,}5$$

$$0$$

$$e^{-0{,}04\left(\frac{1}{52}\right)}(0{,}488)(0{,}731) = 0{,}356$$

$$\delta_c = \frac{0{,}731}{10{,}72-9{,}33} = 0{,}526$$

$$0 \qquad \delta_c = 0$$

$$0$$

Value of at-the-money put:

$$p = c - S + PV(x) = 0{,}356 - 10 + 10\,e^{-0{,}04\frac{2}{52}} = 0{,}341$$

$$\delta_p = -(1-0{,}526) = -0{,}474$$

Theoretical value of 1 unit of structured product:

$$PV(1050) - 100\times k\times(0{,}356) - 100\times k\times(0{,}341)$$

$$\Leftrightarrow 1048{,}4 - 100\times k\,(0{,}697)$$

Profit margin of 5%: $1048{,}4 - 100\times k \times 0{,}697 = 950$

$$\Rightarrow k = 1{,}41$$

VALUE of AT-THE-MONEY put:

$e^{-0{,}04\frac{1}{12}}\, 0{,}512(0{,}665) = 0{,}341$

$\delta_p = -0{,}474$

$0$, $\delta = 0$ → $0$, $0$

$e^{-0{,}04\frac{1}{52}} \cdot 0{,}512(1{,}3) = 0{,}665$

$\delta_p = -1$ → $0$, $1{,}3$

Position of Bank:

$\Pi = -1M \times 950 = -950M$

$\delta_\Pi = 1M\{141(0{,}526 - 0{,}474)\} = 7{,}332M$

$\Pi = -1M\{1050\, e^{-0{,}04\frac{1}{52}} - 141(0{,}731 + 0)\} = -946{,}1\,M$

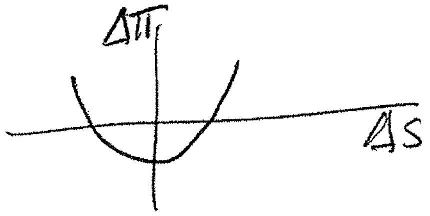
$\delta_\Pi = 1M\{141(1-0)\} = 141\,M$

$\Pi = -1M\{1050 - 141(1{,}5)\} = -838{,}5\,M$

$\delta_\Pi = 141\,M$

Bank is Long on OPTIONS $\Rightarrow$ Positive Gamma

Bank is Long on AT-THE-MONEY OPTIONS $\Rightarrow$ NEGATIVE TETA
SHORT ON RISK-FREE ASSET (LOAN) $\Rightarrow$ NEGATIVE TETA
OVERALL: NEGATIVE TETA



| Week | S(t) | $\delta_\Pi$ | Position in Shares | Δ Position Shares | CF Shares | Risk-Free | Interest | Unhedged Position | Hedged Position |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 10 | +7,332M | −7,332M | −7,332M | +73,32M | +73,32M | | −950M | −950M |
| 1 | 10,72 | +141M | −141M | −133,668M | +1432,92M | +1506,3M | 0,0564M | −946,1M | −946,1M<br>−141M (10,72)<br>+1506,3M<br>= −951,3M |
| 2 | 11,5 | +141M | −141M | 0 | 0 | +1507,5M | 1,16M | −838,5M | −838,5<br>−141M (11,5)<br>+1507,5M<br>= −952,5 |