International Master of Science in Business Economics

Economics of Business and Markets

Problem set 1

Professor Fátima Barros due on December 14, 2012

Exercise 1

The following are the approximate market shares of different brands of soft drinks during the 1980's: Coke: 40%, Pepsi: 30%; 7-Up: 10%; Dr. Pepper: 10%; all other brands: 10%.

a) Compute the Herfindahl index for the soft drink market. Suppose that Pepsi acquired 7-Up. Compute the post-merger Herfindahl index. What assumptions did you make?

b) Federal antitrust agencies would be concerned to see a Herfindahl index of the magnitude you computed in a), and might challenge the merger. Pepsi could respond by offering a different market definition. What market definition might they propose? Why would this change the value of the Herfindahl index?

Exercise 2

Consider a market where two firms, A and B, compete in prices *à la Bertrand* and sell a homogenous product. Both firms know that if they practice a price larger than 9€ no consumer will buy the product; if the price is equal or smaller than 9€, each consumer will buy one unit. Currently, each firm has a constant marginal cost equal to 3€.

Suppose there are 10.000 potential consumers in this market. If both firms practice the same price, each one will cover half of the market. Both firms have enough production capacity to serve the entire market on their own.

a) What is the market equilibrium in terms of price and quantities and profits.

b) Suppose firm A may adopt a new technology that reduces its marginal cost to 2€. How much is A willing to pay for that new technology?

Assume that both firms adopted the new technology mentioned in question b). Since their profit margins have been squeezed through price competition, firms decided to collude.

c) Define trigger strategies that will allow firms to sustain a collusion agreement.

d) For what values of the discount rate i will the collusive solution be the equilibrium of the infinitely repeated game? (Remember that $\delta = 1/(1+i)$ is the discount factor)

Exercise 3

Two firms compete in the same market *à la Cournot*. The inverse market demand is given by $P = 1 - Q$ and both firms have negligible costs.

a) Compute the equilibrium price and the quantities and profits of each firm.

In the next table you can find the two firms' profits when they play three different strategies: strategy L corresponds to the firm's production level in a cartel situation; M is the strategy adopted by a firm when it plays in a non-cooperative setting; and finally H corresponds to the situation where one of the firms unilaterally deviates from the cartel equilibrium.

Firm 1 \ Firm 2	$q_2 = L = 1/4$	$q_2 = M = 1/3$	$q_2 = H = 3/8$
$q_1 = L = 1/4$	0.125 0.125	0.104 0.139	0.094 0.141
$q_1 = M = 1/3$	0.139 0.104	0.11 0.11	0.097 0.109
$q_1 = H = 3/8$	0.141 0.094	0.109 0.097	0.094 0.094

b) Which is the Nash equilibrium of this game assuming that it is played only once? Compare it to your solution from question a). Explain.

c) Assume now that the game is repeated an infinite number of periods. In each period t both firms observe the last period rival's production and each firm j chooses the production level $q_j(t) \in \{L, M, H\}$ as $j = 1, 2$ and $t = 1, 2, \ldots$.
Define a trigger strategy on quantities that will lead to the collusive equilibrium (L, L) in each period. For which discount rate i is this trigger strategy a Nash equilibrium of this non-cooperative repeated game?

Exercise 4

Deuce Hardware Stores are running the following pricing scheme. Anyone who buys $50 or more worth of goods and pays for these commodities in cash, obtains a 20% rebate. However, the rebate is to be not given as cash but, instead, is paid out with "Deuce money". This is comprised of coupons with dollar values indicating the value at which the coupons may be used to purchase goods at a Deuce store at a later date. What kinds of price discrimination do you think this scheme employs?

(in *Industrial Organization: Contemporary Theory and Practice*, Peppal, Richards and Norman, South-Western College Publishing, 1999)

Exercise 5

In the 1960´s Xerox was the major supplier of copying machines. However, the company typically did not sell its machines but only leased them. The lease contract specified a flat charge of $25 per month and a user charge of 3.5 cents per copy, with a required minimum of 2000 copies per month. Thus, a customer who made exactly this minimum number of copies would pay an annual charge of $1,140, while a customer who made 20.000 copies per month would pay an annual rental fee of $8.700.

Source: E. Blackstone, "Restrictive Pratices in the Marketing of Electrofax Copying Machines and Supplies: The SCM Case", Journal of Industrial Economics, 23 (May 1973)

Explain Xerox's strategy of leasing copying machines instead of selling them. Is this a case of price discrimination?

Exercise 6

Suppose consumers' preferences for movies are uniformly distributed along a [0,2] segment, where 0 represents movies for children and 2 represents the most violent movies. Consider that there exist only two cinemas. Each cinema exhibits a certain variety of movies, corresponding to the preference points {0 and 2}: the two cinemas are 21st Century Child (location 0) and Horror Pictures (location 2). The cinemas in this market have negligible marginal costs.

If a consumer watches a movie that does not exactly correspond to his preference, he incurs in a *disutility* $2(v-x)^2$, measured in Euros, which can be interpreted as a 'transportation cost' to consume a different variety. Assume that the market must always be covered, that each consumer will watch one movie and that the reservation price for each one is very high. There are N consumers in this market.

a) Find the reaction functions in terms of prices for each cinema. Compute the equilibrium prices and profits.

Concerned with the content of the latest movies for children, the Government nationalized 21st Century Child. Since the Ministry of Education considers that movies for children should be a public good, 21st Century Child would exhibit its movies free of charge.

b) Compute the equilibrium in terms of prices and profits for the two cinemas under these circumstances. Explain why Horror Pictures would still be able to make profits despite the existence of movies that are offered for free.

c) Imagine that the 'transportation cost' decreases by a certain percentage α, such that it is given by $(1-\alpha)2(v-x)^2$, where $0<\alpha<1$. What would be the effect on Horror Pictures' market share and profits?

After some time, the Government realized that it had more important tasks to perform than entering into the cinema business, and privatized again 21st Century Child.

d) Compute the new equilibrium in terms of prices and quantities for each cinema (assume $\alpha = 0$). Determine the Herfindahl index for this market, and compare it with the case presented in question b). Would you say that it reflects here appropriately the level of price competition? Justify.

Exercise 7

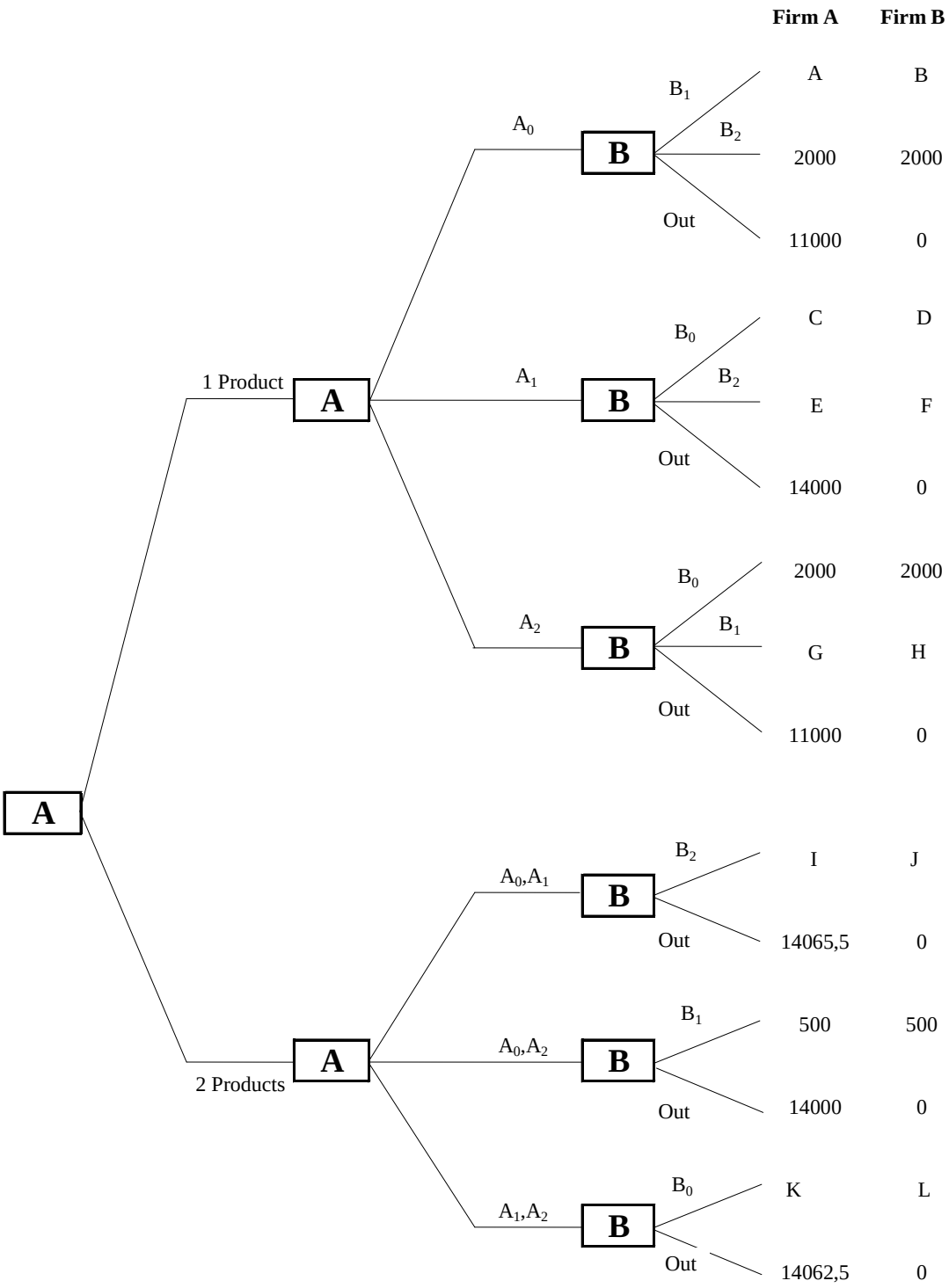
Suppose that in a certain region there is a single breakfast cereals producer. Consumers in this region have preferences that are uniformly distributed along a (0,2) segment, where 0 represents the sweetest type of cereals and 2 represents the healthiest type of cereals. In this market there are 1000 consumers that will buy one unit of the product if and only if the total price is lower than their reservation price that is 15 Euros. Due to technological restrictions firms can only introduce products at points {0, 1or 2} of the segment. The firms in this market have negligible marginal costs and they want to cover the entire market. Each consumer buys a single unit and incurs in a *disutility cost* that is equal to $(v-x)^2$, where *v* is the firm's location in terms of its product variety and *x* is the consumer location in terms of its preferences.

Two firms are considering entering in this market. Assume that firm A is the first deciding to enter and deciding about the product variety she will sell. Firm A can choose to sell one or two products. After firm A entry, firm B decides if she wants to enter or to stay out and, in case of entry, firm B must choose the variety of its product. In the picture you can find the profits that both firms get in all possible cases.

a) Compute the necessary price equilibria in order to find the values of the twelve parameters (A, B, … , L) in the figure below. (**Hint:** Exploit the symmetry of some of these problems to avoid unnecessary computations)

b) Which is the Nash Equilibrium of this game?

c) Suppose that firm B must incur an annual fixed cost of $600. Would you change your answer to the previous question? Why?


Firm A
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1 Product
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14065,5
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Solutions

Exercise 1

a)

Before the merger: $HI = 0.4^2 + 0.3^2 + 0.1^2 + 0.1^2 = 0.27$

After the merger: $HI = 0.4^2 + 0.4^2 + 0.1^2 = 0.33$

Assumptions:

- The merger results in the simple sum of the market shares.
- All other brands are so small that they can be ignored.

b)

Topics for the answer:

- Pepsi may propose to use the beverage sector as whole or insist on including other international players before computing the index.
- If defined differently, the market could appear more competitive.

Exercise 2

a)

$P_A = P_B = 3$, $Q_A = Q_B = 5000$, $\pi_A = \pi_B = 0$

b)

With the new technology: $P_A = 3$, $Q_A = 10000$, $Q_B = 0$, $\pi_A = 10000$

Firm A is willing to pay 10000.

c)

1st period: choose $P = 9$

2nd period: choose $P = 9$ if the other firm chose $P = 9$ in all previous periods; choose $P = 2$ otherwise.

d)

Collusion: $P = 9 \Rightarrow \pi_i = (9-2) \cdot 5000 = 35000$

Deviation: $P = 8.999... \approx 9 \Rightarrow \pi_i = (9-2) \cdot 10000 = 70000$

Nash equilibrium: $P = 2 \Rightarrow \pi_i = (2-2) \cdot 5000 = 0$

$$\text{Collusion} \geq \text{Deviation}$$

$$35000 + \frac{35000}{r} \geq 70000 + \frac{0}{r}$$

$$\Leftrightarrow r \leq 100\%$$

Exercise 3

a)

$Max\ \pi_1 = (1 - q_1 - q_2)q_1$ $\qquad$ $Max\ \pi_2 = (1 - q_1 - q_2)q_2$

$FOC: \frac{\partial \pi_1}{\partial q_1} = 0 \Leftrightarrow q_1 = \frac{1 - q_2}{2}$ $\qquad$ $FOC: \frac{\partial \pi_2}{\partial q_2} = 0 \Leftrightarrow q_2 = \frac{1 - q_1}{2}$

Equilibrium:

$$\begin{cases} q_1 = \frac{1 - q_2}{2} \\ q_2 = \frac{1 - q_1}{2} \end{cases} \Leftrightarrow q_1 = q_2 = \frac{1}{3} \Rightarrow P = \frac{1}{3} \Rightarrow \pi_1 = \pi_2 = \frac{1}{9}$$

b)

The Nash equilibrium is (M,M), with a payoff of (0.11 , 0.11). It is identical to the solution from a) [except for the rounding]. This result was expected since in the Cournot equilibrium each firm is on its reaction function, '*choosing the best response given the decision of the other player*'. Hence, it must be a Nash equilibrium.

c)

Collusion: $q_i = 0.25 \Rightarrow P = 0.5 \Rightarrow \pi_i = 0.125$

Deviation: $\begin{cases} q_i = 0.375 \\ q_j = 0.25 \end{cases} \Rightarrow P = 0.375 \Rightarrow \pi_i = \frac{9}{64} = 0.140625$

Nash equilibrium: $q_i = \frac{1}{3} \Rightarrow P = \frac{1}{3} \Rightarrow \pi_i = \frac{1}{9}$

$$\text{Collusion} \geq \text{Deviation}$$

$$0.125 + \frac{0.125}{r} \geq 0.140625 + \frac{\left(1/9\right)}{r}$$

$$\Leftrightarrow r \leq \frac{8}{9} = 88.(8)\%$$

Exercise 4

Exercise 5

Yes, it's a two-part tariff scheme designed to extract more consumer surplus (here, the '*Fixed fee*' is equal to the sum of the flat charge and the minimum amount of copies). With this lease scheme Xerox is able to do price discrimination between large and small users (larger users pay a lower average price). Price discrimination would not be possible if Xerox sells the copying machines instead of leasing them.

Exercise 6

a)

Indifferent consumer

$$x = \frac{P_H - P_C + 8}{8}$$

Cinema 21st Century Child

$$\underset{P_C}{Max}\ \pi_C = P_C\left[x - 0\right] \cdot \frac{N}{2} = P_C\left[\frac{P_H - P_C + 8}{8}\right] \cdot \frac{N}{2}$$

$$FOC: \frac{\partial \pi_C}{\partial P_C} = 0 \Leftrightarrow P_C = \frac{P_H + 8}{2}$$

Cinema Horror Pictures

$$\underset{P_H}{Max}\ \pi_H = P_H\left[2 - x\right] \cdot \frac{N}{2} = P_C\left[\frac{P_C - P_H + 8}{8}\right] \cdot \frac{N}{2}$$

$$FOC: \frac{\partial \pi_H}{\partial P_H} = 0 \Leftrightarrow P_H = \frac{P_C + 8}{2}$$

Equilibrium:

$$\begin{cases} P_C = \frac{P_H + 8}{2} \\ P_H = \frac{P_C + 8}{2} \end{cases} \Rightarrow P_C = P_H = 8 \Rightarrow \pi_C = \pi_H = 4N$$

b)

Indifferent consumer

$$0 + 2(0 - x)^2 = P_H + 2(2 - x)^2 \Rightarrow x = \frac{P_H + 8}{8}$$

Cinema Horror Pictures

$$\underset{P_H}{Max}\ \pi_H = P_H\left[2 - x\right] \cdot \frac{N}{2} = P_C\left[\frac{8 - P_H}{8}\right] \cdot \frac{N}{2}$$

$$FOC: \frac{\partial \pi_H}{\partial P_H} = 0 \Leftrightarrow P_H = 4$$

$$\Rightarrow x = 1.5$$

$$\begin{cases} \pi_C = 0 \\ \pi_H = N \end{cases}$$

Due to the quadratic disutility, a consumer '*close*' to Horror Pictures prefers to pay something for the movie than to watch for free the movie from 21st Century Child.

c)
Indifferent consumer

$$0+2(1-\alpha)(0-x)^2 = P_H + 2(1-\alpha)(2-x)^2 \Rightarrow x = \frac{P_H + (1-\alpha)8}{(1-\alpha)8}$$

Cinema Horror Pictures

$$\underset{P_H}{Max}\ \pi_H = P_H[2-x]\cdot\frac{N}{2} = P_C\left[\frac{(1-\alpha)8 - P_H}{(1-\alpha)8}\right]\cdot\frac{N}{2}$$

$$FOC:\ \frac{\partial \pi_H}{\partial P_H} = 0 \Leftrightarrow P_H = (1-\alpha)4$$

$$\Rightarrow x = 1.5$$

$$\begin{cases}\pi_C = 0\\ \pi_H = (1-\alpha)N\end{cases}$$

If the transportation cost decreases by α, the market share of Horror Pictures stays constant, and the price decreases by α. Hence, the profit also decreases by α.

d)
Herfindahl index in question b): $HI = 0.75^2 + 0.25^2 = 0.625$
Herfindahl index in question d) [= a)]: $HI = 0.5^2 + 0.5^2 = 0.5$

From b) to d) the Herfindahl index decreased. However, the price level has increased (prices are now equal to 8; previously they were equal to 4 and 0).
In this case the Herfindahl index does not reflect the price competition/price level.

Exercise 7

a)
Parameters A and B?

Indifferent consumer

$$P_A + (0-x)^2 = P_B + (1-x)^2 \Rightarrow x = \frac{1}{2} + \frac{P_B - P_A}{2}$$

Firm A

$$\underset{P_A}{Max}\ \pi_A = P_A[x-0]500 = 500\cdot P_A\left[\frac{1}{2} + \frac{P_B - P_A}{2}\right]$$

$$FOC:\qquad 500\cdot\left[\frac{1}{2} - P_A + \frac{P_B}{2}\right] = 0 \Leftrightarrow P_A = \frac{1+P_B}{2}$$

Firm B:

$$\underset{P_B}{Max}\ \pi_B = P_B\left[(2-x)\right]500 = 500\cdot P_B\left[\frac{P_A - P_B}{2} + \frac{3}{2}\right]$$

$$FOC: \qquad 500\cdot\left[\frac{3}{2} - P_B + \frac{P_A}{2}\right] = 0 \Leftrightarrow P_B = \frac{3+P_A}{2}$$

Equilibrium:

$$\begin{cases} \frac{1}{2} - P_A + \frac{P_B}{2} = 0 \\ \frac{3}{2} - P_B + \frac{P_A}{2} = 0 \end{cases} \Leftrightarrow \begin{cases} P_A = \frac{5}{3} \\ P_B = \frac{7}{3} \end{cases}$$

$$\Rightarrow A = \pi_A = 500\cdot\frac{5}{3}\left[\frac{1}{2} + \frac{7/3 - 5/3}{2}\right] = \frac{6250}{9} = 694.(4)$$

$$\Rightarrow B = \pi_B = 500\cdot\frac{7}{3}\left[\frac{5/3 - 7/3}{2} + \frac{3}{2}\right] = \frac{12250}{9} = 1361.(1)$$

By symmetry:

- A=D=F=G
- B=C=E=H

Parameters I and J?

Two indifferent consumers

$$P_{A0} + (0 - x_1)^2 = P_{A1} + (1 - x_1)^2 \Rightarrow x_1 = \frac{P_{A1} - P_{A0} + 1}{2}$$

$$P_{A1} + (1 - x_2)^2 = P_B + (2 - x_2)^2 \Rightarrow x_2 = \frac{P_B - P_{A1} + 3}{2}$$

Firm A

$$\underset{P_{A0},P_{A1}}{Max}\ \pi_A = \left[P_{A0}(x_1 - 0) + P_{A1}(x_2 - x_1)\right]500 = \left[P_{A0}\left(\frac{P_{A1} - P_{A0} + 1}{2}\right) + P_{A1}\left(\frac{P_B - 2P_{A1} + P_{A0} + 2}{2}\right)\right]500$$

FOC

$$\begin{cases} \frac{\partial \pi_A}{\partial P_{A0}} = 0 \\ \frac{\partial \pi_A}{\partial P_{A1}} = 0 \end{cases} \Leftrightarrow \ldots \Leftrightarrow \begin{cases} P_{A1} - P_{A0} + 0.5 = 0 \\ P_B - 4P_{A1} + 2P_{A0} + 2 = 0 \end{cases}$$

Firm B:

$$\underset{P_B}{Max}\ \pi_B = P_B\left[(2 - x_2)\right]500 = 500 \cdot P_B\left[\frac{-P_B + P_{A1} + 1}{2}\right]$$

FOC

$$\frac{\partial \pi_B}{\partial P_B} = 0 \Rightarrow -2P_B + P_{A1} + 1 = 0$$

Equilibrium:

$$\begin{cases} P_{A1} - P_{A0} + 0.5 = 0 \\ P_B - 4P_{A1} + 2P_{A0} + 2 = 0 \\ -2P_B + P_{A1} + 1 = 0 \end{cases} \Leftrightarrow \begin{cases} P_{A0} = \frac{17}{6} \\ P_{A1} = \frac{7}{3} \\ P_B = \frac{5}{3} \end{cases}$$

$$\begin{cases} x_1 = 0.25 \\ x_2 = \frac{7}{6} \end{cases}$$

$$\Rightarrow I = \pi_A = \frac{25625}{18} = 1423.6(1)$$

$$\Rightarrow J = \pi_B = \frac{6250}{9} = 694.(4)$$

By symmetry:

- I=K
- J=L

b)

Nash equilibrium is given by the case where firm A enters with just one product on one of the extremes (either 0 or 2) and firm B occupies the other extreme.

c)

In the new Nash equilibrium firm A would enter with two products on positions A0 and A2 and firm B would stay out of the market.