Economics of Business and Markets

International MSc. In Bussiness Administration

Solutions of Problem Set 1

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Exercise 1

a)

	C_4 – ratio	Herfindahl index
Industry A	70%	0,2698
Industry B	76%	0,166

- Industry B looks more competitive
- In this case, the Herfindahl index gives a better indication of the concentration, since it is able to capture the dispersion of market shares

b)

Industry A's new Herfindahl index = 0,2992

Exercise 2

Topics for the answer:

- Defining the relevant market (substitutability, geographic dispersion, product classes).
- Price competition and concentration (Bertrand paradox, product differentiation).
- Comparison of Herfindahl index with C_K – ratio

Exercise 3

a) Topics for the answer:

- Aeronautics and Chocolate industries show much higher concentration ratios than Cement and Wine, both with the C_4 and with the Herfindahl-index
- While Aeronautics has the highest value for the C_4, the Chocolate industry has the highest Herfindahl value. This means that although the 4 biggest firms have a

bigger market share in the Aeronautics industry, there is a larger asymmetry among the companies in the Chocolate industry.

Characteristics and limitations of the proposed indexes:

C_4

+ gives an idea of the weight of the four biggest firms
+ allows to analyze the evolution of the joint market share of those firms
- doesn´t detect variations among the biggest four firms
- does not capture any information from the other firms

Herfindahl

+ captures asymmetries in the distribution of the market shares
- simplistic measure, since it summarizes all the information into one value

Furthermore, both measures are vulnerable to the following issues:

- Impact of foreign imports
- The definition of the relevant market
- Industry definitions and product classes

b) The inclusion of the penetration of foreign firms is likely to affect the concentration ratios for the cement and the aeronautics industry differently.

Due to the high transportation costs in the cement industry, the relevant market is usually on a local basis. Therefore, foreign imports into the US should be almost absent (except at the boarders) and the inclusion should have a negligible impact. However, for the aeronautics industry, the transportation costs can be considered low when compared to the value of the products. Foreign imports are likely to have a considerable presence in the US market, and its inclusion in the national statistics should lower the concentration ratios.

Exercise 4

a) Nash Equilibrium: (Q_{NC}, Q_{NC}). It does not correspond to the collusive equilibrium, which is (Q_C, Q_C).

b) Each firm defines the following *trigger* strategy:

- 1st period: produce Q_C.
- Following periods: produce Q_C if the outcome in all the previous periods has been (Q_C, Q_C); produce Q_{NC} otherwise.

In order for firm 1 to follow its own strategy the following condition needs to hold:

$$7+7\delta+7\delta^2+\ldots \geq 8+3\delta+3\delta^2+\ldots \Leftrightarrow \delta \geq 0{,}2$$

In order for firm 2 to follow its own strategy the following condition needs to hold:

$$6+6\delta+6\delta^2+\ldots \geq 10+4\delta+4\delta^2+\ldots \Leftrightarrow \delta \geq 2/3$$

- ⇨ Final answer: $\delta \geq 2/3$

Exercise 5

a) One firm

Firm A will locate itself in the center of the market (location 1).
Price will be set at the level where it is binding for the consumers at the extremes:

$$P+(1-2)^2=10 \Leftrightarrow P=9$$

$$\Pi_A = 9\cdot N = 9\cdot 2million = 18million$$

b) Two firms

Firm A will locate itself in one of the extremes, and firm B will choose the other extreme. Let's assume that firm A moves to location 0.

Indifferent consumer:

$$P_A+(0-x)^2 = P_B+(2-x)^2 \Leftrightarrow x=\frac{P_B-P_A+4}{4}$$

Each firm will try to maximize its own profits, taking the competitor´s price as given.

Firm A:

$$\underset{P_A}{Max} \qquad \Pi_A = P_A[x-0]\cdot N = P_A\left[\frac{P_B-P_A+4}{4}\right]\cdot N$$

$$FOC \qquad \frac{\partial \Pi_A}{\partial P_A}=0 \Leftrightarrow P_A=\frac{P_B+4}{2}$$

Firm B:

$$\underset{P_B}{Max} \qquad \Pi_B = P_B\cdot[2-x]\cdot N = P_B\cdot\left[\frac{P_A-P_B+4}{4}\right]\cdot N$$

$$FOC \qquad \frac{\partial \Pi_B}{\partial P_B}=0 \Leftrightarrow P_B=\frac{P_A+4}{2}$$

The values for each of the prices result from solving the system with the two reaction functions:

$$\Leftrightarrow \begin{cases} P_A = 4 \\ P_B = 4 \end{cases} \Rightarrow x = 1$$

These prices result in the following profits for the two firms:

$$\Pi_A = \Pi_B = 4 \cdot 0{,}5N = 4 million$$

c) Probability of firm B entering the market= 35%

In order to answer this question, one needs to analyse two more situations:
Situation 1 – firm A did not anticipate the entry of firm B, and firm B enters.
Situation 2 – firm A did anticipate the entry of firm B, but firm firm B does not enter.

<u>Situation 1</u>

Doing the usual computations, with firm A at the center, and firm B at one of the extremes:

$$\begin{cases} P_A = 7/3 \\ P_B = 5/3 \end{cases} \Rightarrow x = 5/6$$

$$\Pi_A = 2{,}72 million$$

$$\Pi_B = 1{,}39 million$$

Hence, the expected profit for firm A to go to the center of the market is:

$$E[\Pi_A] = 0{,}65 \cdot 18 million + 0{,}35 \cdot 2{,}72 million = 12{,}7 million$$

<u>Situation 2</u>

If firm B does not enter, firm A, at one of the extremes, will have to practice a price such that it covers the entire market and maximizes its profits simultaneously.
That price will be equal to

$$P + (2-0)^2 = 10 \Leftrightarrow P = 6$$

$$\Pi_A = 6 \cdot 2 million = 12 million$$

Hence, the expected profit for firm A to go to the extreme of the market is:

$$E[\Pi_A] = 0{,}65 \cdot 12 million + 0{,}35 \cdot 4 million = 9{,}2 million$$

Firm A will decide to go to the center of the market.

d) Cartel

If the firms form a cartel, both will practice prices equal to 9. Each firm will make a profit of 9 million.
However, each firm will have an incentive to deviate from this equilibrium. You can check this with the reaction functions found in question b).

For instance, firm A will have an incentive to practice a price lower than 9 if it expects that firm B respects the agreement:

$$P_A = \frac{P_B + 4}{2} = \frac{9+4}{2} = 6{,}5$$

Exercise 6

a) One firm

If firm A believes that there will be no second firm, it will choose to introduce only one product at position 1. The price will be 11.

b) Two firms – firm A produces one product for segment 1 (familiar)

Since entering yields positive profits for firm B, it should enter at one of the extremes, for example segment 0 (health).

Indifferent consumer:

$$P_A + (1-x)^2 = P_B + (0-x)^2 \Leftrightarrow x = \frac{P_A - P_B + 1}{2}$$

Each firm will try to maximize its own profits, taking the competitor´s price as given.

Firm A:

$$\underset{P_A}{Max} \quad \Pi_A = P_A[2-x]\cdot N = P_A\left[\frac{P_B - P_A + 3}{2}\right]\cdot N$$

$$FOC \quad \frac{\partial \Pi_A}{\partial P_A} = 0 \Leftrightarrow P_A = \frac{P_B + 3}{2}$$

Firm B:

$$\underset{P_B}{Max} \quad \Pi_B = P_B \cdot [x-0]\cdot N = P_B \cdot \left[\frac{P_A - P_B + 1}{2}\right]\cdot N$$

$$FOC \quad \frac{\partial \Pi_B}{\partial P_B} = 0 \Leftrightarrow P_B = \frac{P_A + 1}{2}$$

The values for each of the prices result from solving the system with the two reaction functions:

$$\Leftrightarrow \begin{cases} P_A = 7/3 \\ P_B = 5/3 \end{cases} \Rightarrow x = 5/6$$

These prices result in the following profits for the two firms:

$\Pi_A = 1{,}52\, million$

$\Pi_B = 0{,}19\, million$

c) Two firms – firm A produces two products, at locations 0 and 1

Since entering yields positive profits for firm B, it should enter at segment 2 (children).

There will be two indifferent consumers:

- Between positions 0 and 1:

$$P_{A0} + (0 - x_1)^2 = P_{A1} + (1 - x_1)^2 \Leftrightarrow x_1 = \frac{P_{A1} - P_{A0} + 1}{2}$$

- Between positions 1 and 2:

$$P_{A1} + (1 - x_2)^2 = P_B + (2 - x_2)^2 \Leftrightarrow x_2 = \frac{P_B - P_{A1} + 3}{2}$$

Each firm will try to maximize its own profits, taking the competitor´s price as given.

Firm A:

$$\underset{P_{A0}, P_{A1}}{Max} \qquad \Pi_A = P_{A0}[x_1 - 0] \cdot N + P_{A1}[x_2 - x_1] \cdot N$$

$$= P_{A0}\left[\frac{P_{A1} - P_{A0} + 1}{2}\right] \cdot N + P_{A1}\left[\frac{P_B - 2P_{A1} + P_{A0} + 2}{2}\right] \cdot N$$

$$FOC \qquad \begin{cases} \dfrac{\partial \Pi_A}{\partial P_{A0}} = 0 \\ \dfrac{\partial \Pi_A}{\partial P_{A1}} = 0 \end{cases} \Leftrightarrow \begin{cases} -2P_{A0} + 2P_{A1} + 1 = 0 \\ 2P_{A0} - 4P_{A1} + P_B + 2 = 0 \end{cases}$$

Firm B:

$$\underset{P_B}{Max} \qquad \Pi_B = P_B \cdot [2 - x_2] \cdot N = P_B \cdot \left[\frac{P_B - P_{A1} + 3}{2}\right] \cdot N$$

$$FOC \qquad \frac{\partial \Pi_B}{\partial P_B} = 0 \Leftrightarrow P_B = \frac{P_{A1} + 1}{2}$$

The values for each of the prices result from solving the system of the three reaction functions:

$$\Leftrightarrow \begin{cases} P_{A0} = 17/6 \\ P_{A1} = 7/3 \\ P_B = 5/3 \end{cases} \Rightarrow \begin{cases} x_1 = 1/4 \\ x_2 = 7/6 \end{cases}$$

These prices result in the following profits for the two firms:

$\Pi_A = 0{,}45\, million$

$\Pi_B = 0{,}19\, million$

d) Nash equilibrium

Firm A will choose to introduce two products, at locations A0 and A2. Firm B will decide to not enter the market

Exercise 7

a) Equilibrium

As usual, the indifferent consumer needs to be computed (we will consider that firm A moves to Faro and firm S to Vigo):

$$P_A + (0-x)^2 = P_S + (3-x)^2 \Leftrightarrow x = \frac{P_B - P_A + 9}{6}$$

Each firm will try to maximize its own profits, taking the competitor´s price as given. This problem is summarized in the following mathematical problem.

Firm Alca-Tell:

$$\max_{P_A} \quad \Pi_A = P_A[x-1]\cdot N = P_A\left[\frac{P_S - P_A + 3}{6}\right]\cdot N$$

$$FOC \quad \frac{\partial \Pi_A}{\partial P_A} = 0 \Leftrightarrow P_A = \frac{P_S + 3}{2}$$

Firm Sam-Sing:

$$\max_{P_S} \quad \Pi_S = P_S \cdot [2-x]\cdot N = P_S \cdot \left[\frac{P_A - P_S + 3}{6}\right]\cdot N$$

$$FOC \quad \frac{\partial \Pi_S}{\partial P_S} = 0 \Leftrightarrow P_S = \frac{P_A + 3}{2}$$

The values for each of the prices result from solving the system with the two optimal conditions that come from the maximization problems:

$$\Leftrightarrow \begin{cases} P_A = 3 \\ P_S = 3 \end{cases} \Rightarrow x = \left[\frac{4 + P_A - P_S}{4}\right] = 1{,}5$$

These prices result in the following profits for the two firms (although x=1,5, each firm just gets 0,5N consumers, since the market just goes from location 1 to location 2):

$$\Pi_A = \Pi_S = 3\cdot 0{,}5\cdot N = 1{,}5\cdot N$$

b) Collusive equilibrium

The best scenario for the cartel would be to have one firm in Lisbon (e.g. Alca-Tell) and the other in Porto (e.g. Sam-Sing).

Prices for both firms would be $P = 5{,}75$ (reservation price of 6€ minus the transportation cost).
Profits would be $\Pi = 5{,}75 \cdot 0{,}5 \cdot N = 2{,}875N$

c) Nash equilibrium in the repeated game

Firms will not deviate if:

$$2{,}875N + \frac{2{,}875N}{r} > 4{,}75N + \frac{1{,}5N}{r} \Rightarrow r < \frac{11}{15} = 73{,}(3)\%$$